Investigating curricular effect on the teaching and learning of mathematics in a cultural context: Theoretical and methodological considerations

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ABSTRACT

This first article of the special issue presents theoretical and methodological considerations about longitudinally investigating curricular effects on the teaching and learning of mathematics in a cultural context. In particular, in this article, we discuss seven issues related to the investigation of curricular effects on the teaching and learning of mathematics: (1) the need to investigate longitudinally the effects of curriculum, (2) the nature of the reform and non-reform curricula, (3) multiple representations of curriculum use, (4) assessment of classroom instruction, (5) student diversity and achievement gaps, (6) multiple measures of student mathematics achievement, and (7) multi-level analysis of student achievement data. This article not only serves as an introduction to the special issue, but most importantly, also provides a theoretical and methodological context for the two longitudinal projects.

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China1 and the United States of America have quite different cultural traditions, such as philosophical and epistemological traditions (Nisbett, 2003) and the cultural preference for individualism or for collectivism (Ho & Chiu, 1994; Yu & Yang, 1994). Cultural traditions can serve as normative frameworks for studying teaching and learning in school. A number of researchers have shown the cultural differences in the teaching and learning of mathematics between China and US or, more generally, between Asian countries and the US (e.g., Cai, 1995; Fan, Wong, Cai, & Li, 2004; Schleppenbach, Perry, Miller, Sims, & Fang, 2007; Schmidt et al., 2002; Stevenson et al., 1990; Stevenson & Stigler, 1992; Stigler & Hiebert, 1999).

Although there are enormous differences in terms of cultural traditions between China and the US, the two countries have been undertaking similar mathematics education reforms in recent years. The overarching goals of the reforms in the two countries are to improve students’ learning of mathematics and to nurture students’ innovation and creativity (Cai et al., this issue; Ni et al., this issue). The world has been changing dramatically, and these changes are happening much faster than many of us have anticipated. Today, possessing a large amount of knowledge and information is not sufficient. Instead, in this continually changing world, the most important qualities that we can help our students develop are the abilities to think independently and critically, to learn, and to be creative. In his best-selling book, The World Is Flat, Friedman (2005) pointed out that “there may be a limit to the number of good factory jobs in the world, but there is no limit

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1 The reform curriculum was implemented in People's Republic of China. Thus, in this special issue, all instances of the word 'China' refer to the People's Republic of China.

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to the number of idea-generated jobs in the world” (p. 230). K-16 education, in general, and mathematics education, in particular, has the responsibility for nurturing students’ creativity and critical thinking skills for their life-long learning.

Not only are there similar overarching goals for mathematics education reform in both China and US, but also there is a similar mechanism for reaching the goals. In both countries, changing the curriculum has been viewed and used as an effective way to change classroom practice and to influence student learning to meet the needs of the ever-changing world. In both China and the United States, advocates of mathematics education reform typically have attempted to change classroom practice, and hence students’ learning, by means of changes in curricula. At the same time, the two countries also have faced similar challenges in implementing the reforms. The question arises: Is the mathematics education reform really effective? This special issue draws on findings from two longitudinal projects—one in China, the other in the US—to address this question and to understand curricular effects on the teaching and learning of mathematics.

The first project, conducted in China, addressed the question, “Has curriculum reform made a difference?” by looking for changes in classroom practice. This project compared the effects of a new, reform2 elementary mathematics curriculum to that of the conventional curriculum on classroom practice and student learning outcomes. The second project—the LieCal Project (Longitudinal Investigation of the Effect of Curriculum on Algebra Learning)—was conducted in the US. This project was designed to investigate not only the ways under which a reform curriculum can or cannot enhance student learning in algebra, but also the characteristics of the curricula that lead to student achievement gains. Both projects looked into changes in classroom practice by examining the nature of classroom instruction, analyzing cognitive features of the instructional tasks implemented in different classrooms and the characteristics of classroom interactions and changes in student learning outcomes.

The Chinese Project and the US LieCal Project share similarities in their designs and data analyses. In particular, both projects address common and critical issues about the teaching and learning in using reform curricula in an international context, including: (1) Does the use of a reform curriculum affect the quality and nature of classroom teaching? (2) Do students gain more in solving problems, as the developers of the reform curricula claim? (3) Do students sacrifice basic mathematical skills with the reform curriculum? (4) To what extent does the use of the reform curriculum improve learning for all students?

In this article, we present some theoretical and methodological considerations about longitudinally investigating curricular effects on the teaching and learning of mathematics in a cultural context. Our goal is not only to provide an introduction to the special issue, but most importantly, to provide a theoretical and methodological context for the two longitudinal projects. In particular, we discuss seven issues: (1) the need to investigate longitudinally the effects of reform curricula, (2) the nature of the reform and non-reform curricula, (3) multiple representations of curriculum use, (4) assessment of classroom instruction, (5) student diversity and achievement gaps, (6) multiple measures of student mathematics achievement, and (7) multi-level analysis of student achievement data.

1. Need for investigating longitudinally the effects of reform curricula

Curriculum not only conveys what mathematics students should learn, but also how mathematics should be taught. Thus, the curriculum serves not only as a means to convey reform ideas, but also as a mechanism for making the reform ideas a reality by enabling teachers to learn mathematics content as well as how to teach it (Ball & Cohen, 1996; Lloyd & Frykholm, 2000; Wang & Paine, 2003).

In both China and the US, there have been ‘mathematics wars’ related to mathematics education reform. The anti-reformers worry that the reform curriculum reduces the level of rigor of the mathematics and that students will not acquire basic mathematical skills. They even worry that the development of students’ higher-order thinking skills, a feature of reform curricula, comes at the expense of the development of basic mathematical skills. The supporters of the reform believe that learning mathematics should go beyond the memorization and acquisition of basic mathematical procedures. Instead, students should be provided opportunities to develop conceptual understanding and higher-order thinking skills. In addition, the supporters of the reform maintain that students will learn basic mathematical skills as they engage in explorations of worthwhile problems. Thus, a core question in the current debate is Will students sacrifice basic mathematical skills if they are taught using the reform mathematics curricula?

Obviously, higher-order thinking skills are important in learning and doing mathematics, but so is fluency in computational procedures and symbolic manipulations. Having higher-order thinking skills does not imply having basic skills and vice versa (Cai, 1995; Hatano, 1990; National Research Council, 2001; Steen, 1999). Therefore, it is reasonable to wonder if students will sacrifice mastery of basic skills by using the reform mathematics curricula. It is also reasonable to wonder if students who use reform curricula can truly develop both conceptual understanding and higher-order thinking skills. In China, other than the project reported in this special issue, there have been no other empirical studies examining the effects of the reform. In the US, even though there have been some studies examining the effects of curricular reform, no comprehensive longitudinal studies have been conducted in urban settings while controlling for variation among teachers and specific classrooms of students. Therefore, the two longitudinal studies reported here provide much-needed empirical evidence about the impact of the recent curriculum innovations on students’ computational proficiency and higher-order thinking skills.

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2 We have chosen to use ‘reform curriculum’ rather than ‘reformed curriculum’ whenever we refer to any curriculum that serves as an alternative to the traditional curriculum.
The two longitudinal projects reported here are more than curriculum evaluation studies. Instead, they are research projects that test fundamental hypotheses about the relations among curriculum, teaching, and learning in an international context. For example, reform curricula can be characterized as teaching mathematics through problem solving, in contrast to the more traditional curricula in both China and the US (see more detailed explanation in Cai, 2003a; Cai, Nie, Moyer, 2010; Cai et al., this issue; Ni et al., this issue; Senk & Thompson, 2003). An intent of the reform curricula is to build students’ understanding of important mathematical ideas, often through explorations of real-world situations and problems. There is growing research evidence that traditional ways of teaching, which involve memorizing and reciting facts, rules, and procedures, with an emphasis on the application of well-rehearsed procedures to solve routine problems, are clearly not adequate (Cai, 2003b, 2010; Hiebert, 1999; Lesh & Zawojewski, 2007). Findings from these two projects provide further evidence of the effectiveness of teaching mathematics through problem solving.

In addition, researchers in previous studies have focused primarily on providing data about student outcomes; they have not, to any great extent, considered reform curricula as venues for studying basic questions about learning (Kilpatrick, 2003). For example, if the development of students’ higher-order thinking skills does not come at the expense of the development of basic mathematical skills when using reform curricula, then how do students develop their basic mathematical skills? Is it the case that students learn algorithms and master basic skills as they engage in explorations of worthwhile, mathematically rich, real-world problems (Putnam, 2003)? The design of the two longitudinal projects presented here helps us better understand the complexity of how basic mathematical skills may be intertwined with conceptual understanding and problem solving and how they support each other.

In the past a few decades, there have been a number of cross-national studies in mathematics education. One trend of these studies has been to explore performance differences and factors causing the performance differences to generate theories about students’ learning. Although the two projects presented here are not comparative in nature, they provide a cultural context for examining the generality about the curricular effects on the teaching and learning of mathematics.

2. Nature of the reform and non-reform curricula

To understand curricular effects on mathematics teaching and learning, we must first address questions such as: What exactly is a reform curriculum? Are reform curricula really different from non-reform curricula? If so, in what way? In the four articles following this one, great effort has been made to provide a clear picture of the differences between the reform and non-reform curricula both in China and the US. To be sure, there are remarkable differences between the reform and non-reform curricula in each country. For example, in the US reform curriculum, a functional approach is used to conceptualize algebraic ideas. However, in the non-reform curriculum, a structural approach is used to present algebraic ideas (Cai et al., 2010). In the new Chinese curriculum, an emphasis is placed on linking school mathematics to real-life applications; such a link does not appear in the traditional curriculum. In addition to focusing on substantive knowledge (i.e., knowledge of mathematics content), the reform Chinese curriculum also pays more attention to how students’ knowledge is constructed and advanced (syntactic knowledge). One result of the emphases on the relation of school mathematics to real life applications and on syntactic knowledge has been recognition of a need to use teaching and learning approaches that are more participatory, exploratory, and constructive. The intent of the developers of the Chinese reform curriculum is to help students not only to acquire basic mathematical knowledge and skills, but more importantly to provide them with the opportunities to reason about evidence and explanation, evaluate knowledge claims, apply knowledge and skills to solve real-life problems, develop interest and confidence in learning and using mathematics, and cultivate a dynamic and exploratory view of mathematics and mathematics learning (see Li & Ni, this issue; Ni et al., this issue for a more detailed discussion on the changes in the new Chinese curriculum).

3. Multiple representations of curriculum use

The two projects clearly followed three levels of curriculum use—intended, implemented, and attained—in their designs (Cai, 2010; National Research Council, 2004). An improved understanding of the importance and complexity of aligning curriculum and instruction was obtained from the lessons of previous failed curriculum innovations and consequently conceptual inquiries about curriculum use. One benefit was the idea of multiple representations of curriculum. According to the research synthesis by Stein, Remillard, and Smith (2007), it takes a chain of transformations for a curriculum to influence classroom instruction and then, eventually, to affect student learning. These transformations include consideration of the curriculum as intended by the designers, as perceived by users, as implemented by teachers in classroom, and as achieved by students. Every layer of the curriculum has its own evaluative criteria for describing, judging, understanding, and explaining a curriculum’s evolution, practices, and outcomes because the factors in play at the different levels vary. Meanwhile, the relationships between the various curricular representations need to be explored and understood because they relate to and constrain one another. Curriculum and instruction jointly affect the nature and level of student learning.

4. Assessment of classroom instruction

The multiple representations of curriculum use signify that the impact of the intended curriculum can only be realized through implementation in the classroom. Therefore, in investigating the curricular effect on the teaching and learning of
mathematics, one needs to take into consideration the fidelity of implementation—the extent to which a curriculum is used in the intended way. Both projects gathered data to examine fidelity of implementation. For example, in the LieCal Project, teacher logs, pre- and post observation interviews, assigned homework, and classroom observations were collected to ascertain the level of each teacher’s implementation of the reform or non-reform curriculum. Using these data, the implementation levels were determined according to (1) the extent of coverage of the curricular materials, (2) the consistency between the intended and implemented curricular goals, (3) the congruence between instruction and curriculum design, and (4) the length of time the curricula were used (National Research Council, 2004). Unfortunately, due to space limitations, discussion of fidelity of implementation is not included in any of the articles. Instead, the articles focus on particular teaching variables to examine curricular effects on the pedagogical features of classroom instruction. These teaching variables include features of instructional tasks and classroom discourse in the Chinese project, and conceptual and procedural emphases and instructional tasks in the US project. In the US project, the achievement data were analyzed by controlling for variation in classroom instruction. However, the Chinese project did not introduce teaching variables into the analysis of student learning outcomes because of the challenge of making use of the multifaceted and descriptive data on the teaching variables (Li & Ni, this issue) to predict student-learning outcomes.

5. Student diversity and achievement gaps

In the United States, various national assessments have shown that there is a large achievement gap between minority and non-minority students (Kloosterman & Lester, 2004, 2007; Silver & Kenney, 2000). In China, recent economic advances have resulted in dramatic increases in income levels and quality of life (Xie, Li, Sun, & Wen, 2008). Unfortunately, these advances have been accompanied by an increase in the gap in educational achievement among people with different socio-economic status (SES) (Xie et al., 2008). Since teaching is a cultural activity, students with different ethnic and SES backgrounds may respond differently to new curricula or instructional practices. Consequently, it is important to investigate the extent to which the use of reform curricula improves the mathematics achievement of all students and helps close achievement gaps between different ethnic groups of students and between students at different SES levels. This is an under-investigated issue in curriculum studies (Schoenfeld, 2002). The two projects reported here investigated this issue, with the Chinese project focusing on SES and the US project focusing on ethnicity.

6. Multiple measures of student achievement

The ultimate goal of educational research and reform and curriculum innovation is to improve students’ learning. Therefore, the direct and most important effect of a curriculum is to examine what students actually learned. The two projects used multiple measures of student achievement. Furthermore, most of the assessment tasks used in both projects came from Cai’s earlier work (Cai, 1995, 2000). Three considerations determined the achievement measures used in both projects: (1) a combination of multiple-choice and open-ended assessment tasks was used to measure students’ performance; (2) different cognitive components, specifically, the components of Mayer’s (1987) cognitive model (specifically, translation, integration, planning, and computation), were used to design the multiple choice tasks; and (3) in responding to open-ended task, students were required to show their solution processes and provide justifications for their answers. Because of their potential for broad content coverage and objective scoring, their highly reliable format, and their low cost, multiple-choice questions were used to assess whether students have learned the basic knowledge and skills in mathematics. However, multiple-choice questions are more appropriate for measuring procedural knowledge and basic skills than conceptual understanding. Also, it is difficult to infer students’ cognitive processes from their responses to multiple-choice items. Thus, in addition to multiple-choice questions, open-ended tasks were included to assess student achievement in both projects. The open-ended tasks provided a better window into the thinking and reasoning processes involved in students’ problem solving. The use of various types of assessment tasks provided information to answer such question as ‘Does the curricular emphasis on conceptual understanding come at the expense of fluency with basic mathematical skills?’

The use of open-ended assessment tasks makes it possible not only to measure students’ higher-order thinking skills and conceptual understanding, but also to analyze students’ solution strategies, representations, and mathematical justifications (Cai, 1997). Unfortunately, due to space limitations, it was not possible to include in this issue any qualitative analyses associated with students’ solution strategies, representations, and justifications. These analyses can be found in other reports (Cai, Moyer, Wang, & Nie, 2011; Ni, Li, Cai, & Hau, 2009).

7. Multi-level analysis of student achievement data

In both projects, both traditional and modern statistical analysis techniques were used. One of the important differences between traditional and modern statistical analysis techniques is related to the unit of analysis. In traditional analysis (e.g., analysis of covariance) each student in the sample is treated the same. This implies that students from different classes with different teachers in different schools are considered to have had the same experience with a given curriculum program. However, the reality is that students from different classes with different teachers and in different schools are likely to have different experiences with the program. Thus, in investigating curricular effects, it was essential to take into account
variables at both the classroom and student levels (Cai, 2010). As a result, advanced analysis techniques, such as hierarchical linear modeling (HLM), were used to examine the impact of the curriculum on student learning at different levels simultaneously (Raudenbush & Bryk, 2002). Both projects applied HLM techniques to examine curricular effects on student learning, thereby providing a clearer picture of the results.

The seven issues discussed above have posed both conceptual and methodological challenges for investigating curricular effects on teaching and learning and they have shaped our thinking and implementation of the two projects reported in the following four articles. Rich data at both the individual and the classroom level were obtained from the two projects to address the four key questions posed at the beginning of this article.

A pair of reports, one for each project, are presented in this special issue. One report focuses on curricular effects on classroom instruction (‘implemented curriculum’) and the other report focuses on student learning outcomes (‘achieved curriculum’). In particular, the second article reports findings about the impact of curriculum reform on teaching practice in primary mathematics in Mainland China. The focus of the study was on the instructional tasks that were implemented in the classrooms and on the teacher and student interaction.

The third article provides evidence that the kinds of instruction that transpires in classrooms using reform-based and traditional mathematics curricula in the United States. The focus of the study was on the examination of different types of lesson activities, the use of three types of teaching materials, and the nature of instruction.

The fourth article is a report of a study of the curricular effects on student mathematics achievement by following two groups of students from fifth to sixth grade who were taught either the reform curriculum or the conventional curriculum in Mainland China. The results showed overall improved performance among all the students over time on computation, routine problem solving, and complex problem solving. There were differentiated patterns of performance between the groups. On the initial assessment, the reform group performed better than the non-reform group on calculation and complex problem solving. There was no significant difference in growth rate between the groups on the achievement measures except that the non-reform group progressed at a faster pace on computation.

The fifth article was an investigation of curricular effects on US middle school students’ learning of algebra using various outcome measures. The results showed that the use of the reform-based curriculum contributed to significantly higher problem-solving growth for all ethnic groups. Moreover, the students’ basic mathematical skills were not sacrificed.

In the last article, we first examined the convergence and divergence of evidence from the two projects relating to curricular effects on the teaching and learning of mathematics. Then we discussed the lessons we learned from the projects. The article concludes with a discussion of new developments underway in these projects that aim to further the efforts to clarify the interdependence between curriculum, teaching and learning.

This special issue provides a unique opportunity to examine the generality of the results from the two distinctive cultural contexts concerning curricular effects on the teaching and learning of mathematics. Collectively, the articles provide an important addition to the literature on mathematics curriculum reform.

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References