Cognition and Instruction

Relations of Instructional Tasks to Teacher-Student Discourse in Mathematics Classrooms of Chinese Primary Schools

Yujing Ni \(^a\), Dehui Zhou \(^a\), Xiaoqing Li \(^a\) & Qiong Li \(^b\)

\(^a\) The Chinese University of Hong Kong

\(^b\) Beijing Normal University

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Relations of Instructional Tasks to Teacher–Student Discourse in Mathematics Classrooms of Chinese Primary Schools

Yujing Ni, Dehui Zhou, and Xiaoqing Li

The Chinese University of Hong Kong

Qiong Li

Beijing Normal University

This study, based on observation of 90 fifth-grade mathematics classes in Chinese elementary schools, examined how the task features, high cognitive demand, multiple representations, and multiple solution methods may relate to classroom discourse. Results indicate that high cognitive demand tasks were associated with teachers’ use higher order questioning. Higher order questioning but not the high cognitive demand tasks themselves generated participatory responses among students. However, when teachers pursued multiple solution methods, they were more inclined to ask memorization and procedural lower order questions than explanatory and analytical higher order questions. Contrary to our hypothesis, high cognitive demand tasks and higher order questions related to teacher authority in evaluating students’ answers whereas neither cognitive demand, multiple representations, nor teachers’ pursuit of multiple solution methods were directly related to teacher–student joint authority in discourse. Implications regarding the relationship between tasks and discourse and instructional practice in a cultural context are discussed.

Instructional tasks and classroom discourse are two of the most important features of teaching and learning in mathematics classrooms. These are key factors that define the links between mathematics teaching and learning (Hiebert & Wearne, 1993; Q. Li & Ni, 2011; Stein, Grover, & Henningson, 1996). The purpose of the present study was to investigate their relationships in Chinese primary mathematics classrooms.

Studies show that the two entities have been clearly defined, but few studies have explicitly examined the relationship between them involving a larger classroom sample and using...
quantitative methods. One of the reasons is that instructional tasks and classroom discourse occur mostly simultaneously and interweave with each other in classroom activity, making it difficult to articulate and test their relations in an explicit way. One assumption, implicit or explicit, about their relationship is that changes in the types of instructional tasks would bring about change in classroom discourse. Some have reasoned that instructional tasks form the basis for students to learn mathematics (Doyle, 1988). Types of mathematics tasks are seen as influencing the nature of students’ engagement with the tasks and with the teacher (Hiebert and Wearne, 1993; J. Liu & Sun, 2002; Schoenfeld, 1992; Stein, Smith, Henningsen, & Silver, 2000).

This assumption illustrates the pivotal role of instructional tasks in shaping the ways of teaching and learning in mathematics classrooms. This was one of the key tenets providing the catalyst for mathematics curriculum reforms since the 1980s: To change the substance of what to teach (i.e., instructional tasks) in order to influence how to teach and how to learn (i.e., classroom discourse; e.g., Ministry of Education, 2001a, 2001b; National Council of Teachers of Mathematics, 1989).

However, the assumption that changing instructional tasks would result in changes in teacher–student discourse in the mathematics classroom needs to be further investigated empirically and clarified conceptually. Primarily, the conditions are not the same for making changes in instructional tasks as they are for changing classroom discourse. For example, Mainland China’s educational system is highly centralized and as such, is a system that can make change in instructional tasks mandatory. This can be accomplished by merely changing curriculum materials. However, the government does not appear to have an effective mechanism for bringing about mandatory changes to classroom discourse, even though the mandatory national curriculum and textbooks contain pedagogical recommendations for creating a more open and dialogic classroom (J. Liu & Sun, 2002). The inherent nature of classroom discourse is greatly determined by the behaviors of individual teachers and students.

Secondly, studies have shown that instructional tasks and classroom discourse are interdependent. Henningsen, Stein, and their colleagues (Henningsen & Stein, 1997; Stein et al., 1996) demonstrated that the nature of instructional tasks have the potential to structure the way that a teacher teaches. These tasks can also influence the ways his or her students think about mathematics (Schoenfeld, 1992). However, many other classroom-based factors, such as classroom norms, teachers’ instructional dispositions, and students’ learning dispositions, can either provide support or alternatively inhibit the implementation of the intended high cognitive demand tasks. High cognitive demand tasks can involve more ambiguity and higher levels of personal risk for both teachers and students (Ball & Wilson, 1996). The mutual constraint of instructional tasks and classroom discourse is expected to be more acute and complex with implementation of high cognitive demand tasks within the context of Chinese culture and traditions. For example, Chinese students were observed to be less willing to take risks to solve mathematics problems (Cai & Cifarelli, 2004) and less tolerant of ambiguity in mathematics classroom (Wang & Murphy, 2004).

The sections that follow describe briefly the changing mathematics teaching and learning in China. These changes, initiated by the 2000s curriculum reforms for the nine-year compulsory education in the country, have provided the background for the present study. The conceptualization of the study is then explicated. Finally, the results of the study are presented and interpreted.
CLASSROOM INSTRUCTION IN THE PRESENT CHINESE ELEMENTARY MATHEMATICS CLASSROOMS

New Millennium Curriculum Reform in Mathematics

Since 1949, China has had a nationally mandated school mathematics curriculum, and there have been at least 10 official syllabi since it was first implemented (X. Ma, 1996; Ministry of Education, 2000, 2001b, 2012). Despite many changes in the Chinese system of teaching mathematics, a unique feature of the Chinese mathematics curriculum has remained. This fundamental philosophy in the curriculum emphasizes “the two basics,” namely basic mathematics concepts and basic mathematics skills. The two basics view stresses the importance of acquisition of foundational knowledge, content and skills rather than the use of creative thinking processes (Leung, 2001).

To complement the school curriculum, mathematics teaching in Chinese classrooms focuses on refined lectures and repeated practice (Zhang, Li, & Tang, 2004). Teachers present their students with well-prepared lessons that include strong teacher control, coherent instruction, in addition to accurate and abstract mathematics (Ni, Chiu, & Cheng, 2010; Schleppenbach, Perry, Miller, Sims, & Fang, 2007; Stevenson & Stigler, 1992; Wang & Murphy, 2004). The main focus of the two basics curriculum for both teachers and students is on the key mathematic concepts, skills, and flexible applications to problems (L. Ma, 1999; Zhang et al., 2004).

On the other hand, the earlier Chinese curricula have been criticized for a stronger focus on substantive knowledge, that is, established facts, concepts and skills, while neglecting syntactic knowledge, or how knowledge is constructed and advanced. This specific curricular orientation gave rise to classroom instruction that attributed less value to the connection between given knowledge and the processes by which knowledge is established. In classroom instruction, the teacher and the textbook were the only knowledge authorities from which the students received “packaged knowledge.” As a result, many Chinese students were not able to apply the skills for learning mathematics to their present and future lives but saw classroom mathematics only as a useful exercise for taking examinations (Y. R. Liu, 2000). Unfortunately, as students moved up to higher grades, their interest and confidence in learning mathematics deteriorated (Ho, 2003; Ni, Li, Li, & Zhang, 2011).

In addition, many Chinese educators believe that repeated practice aids memorization and helps students think about the underlying concepts more deeply (Dhlin & Watkins, 2000; Zhang et al., 2004). Chinese teachers are used to maintaining control by direct teaching to the whole class. Direct teaching helps teachers control the lesson flow to maintain class discipline (especially with 40–60 students per class in many Chinese classrooms), while engaging students in obtaining learning targets (Huang & Leung, 2004). Consequently, Chinese teachers and students generally do not easily tolerate ambiguity in teaching and learning mathematics (Fan, Wong, Cai, & Li, 2004; Lopez-Real, Mok, Leung, & Marton, 2004; Ni et al., 2010; Wang & Murphy, 2004). As Confucian culture assigns content expertise to teachers, Chinese students are also more receptive to the teacher’s dominant role (e.g., Stevenson and Stigler, 1992; Wong, 2004). These limitations of the Chinese system of teaching mathematics have been among those targeted by the most recent curriculum reform (J. Liu & Sun, 2002; Ministry of Education, 2001a, 2001b, 2012).

In 2001, the Chinese government put forth the new curriculum standards, Curriculum Standards for Mathematics Curriculum of Nine-Year Compulsory Education (Ministry of Education,
The new curriculum standards pay more attention to how students can master the syntax of mathematics, namely the procedures and rules to test, verify and extend mathematics knowledge, in addition to the substance of mathematics, established concepts and procedures. The designers of the new curriculum have used two key words, constructive and participatory (J. Liu & Sun, 2002), to highlight this feature of school mathematics to take care of both types of mathematics knowledge. The former refers to shifting the focus of curriculum away from the transmission of knowledge by teachers toward the construction of knowledge by students. The latter refers to the change from receptive learning to participatory learning that supports a more collaborative, exploratory, and hands-on experience for students. In particular, the new curriculum promotes teaching mathematics through problem solving, through which a teacher guides students to observe, analyze, and solve mathematics problems. This process enables students to foster a deeper and more interconnected understanding of concepts, procedures, and principles. Developing the deeper and more interconnected understanding also relies on a more interactive and productive classroom discourse, one that enables students to communicate and to contribute ideas and to enhance their understandings of mathematics as a logical, epistemological, and socially constructed enterprise.

The new mathematics curriculum was first implemented in 2001 and included 38 pilot districts selected from 27 provinces in China. By the end of 2006, the implementation of the new curriculum was mandatory, and all first-grade students were required to use the reformed curriculum across the country. More details about the new mathematics curriculum are discussed in other publications (see J. Liu & Sun, 2002; Y. P. Ma, 2012; Ni et al., 2011).

Prior Work on the Impact of the Curriculum Reform on Instructional Practice in Elementary Mathematics Classrooms in China

Very few empirical studies in China have investigated the impact of the new curriculum on mathematics teaching and learning since its nationwide implementation in 2006. Yu (2003) observed that teaching practices in the classrooms piloting the reform curriculum became more varied and included more active student participation. Y. P. Ma (2005) conducted a survey of the reform classrooms and reported that those teachers encouraged the students to state their views, explain their ideas, and respond to the ideas of their classmates. However, Yu (2003) and Y. P. Ma (2005) also found that teachers piloting the new curriculum had significant problems guiding classroom discussions effectively. The classroom discussions were sometimes little more than teacher centered question-and-answer sessions, where teachers were inclined to pressure students to agree with them; other times, group discussions became situations where some teachers permitted students to do whatever they liked, without purposeful guidance, feedback, or requirements. Similar difficulties were observed with American teachers using reform mathematics curricula that promote productive student engagement in the classroom (Baxter & Williams, 2010; Henningsen & Stein, 1997; Stein, Engle, Smith, & Hughes, 2008; Williams & Baxter, 1996).

In light of the lack of empirical data in this area in China, Ni and her colleagues conducted the project to examine whether or not the new mathematics curriculum influenced classroom practice and consequently improved student learning. The project was carried out from 2005 to 2008 and investigated change in classroom practice and student learning in primary mathematics resulting from recent curriculum reforms (Ni et al., 2011; Ni, Li, Cai, & Hau, 2009).
was situated in Zhen Zhou, a city in the middle part of China with an economic and educational level approximating that of the national average (National Bureau of Statistics, 2003, 2010). The city included six school districts. When the project started, one of the districts had adopted the new mathematics curriculum for all of its elementary schools in 2001, and the other five were still using the conventional curriculum in their elementary schools. One district was selected from the five using the conventional curriculum based on the considerations that it would be comparable with the district using the new curriculum in terms of teacher training credentials, teaching experience, the general achievement levels of the students, and class size. Ten schools were then randomly selected from the school district implementing the new mathematics curriculum, and another 10 schools were randomly selected from the district using the conventional curriculum. Three fifth-grade mathematics teachers were randomly chosen from each of the 20 schools (the elementary schools had four to six classes for each grade level), resulting in a sample of 60 teachers and their students for the project (Ni et al., 2009). Fifth-grade classrooms were recruited for the project because students of the fifth-grade reform classrooms had been using the new curriculum since their first grade (in 2001).

One objective of the project was to investigate whether or not the curriculum reform affected practice in the mathematics classrooms in terms of the nature of instructional tasks implemented and the ways that teachers and students interacted. In the study, each of the 60 teachers and their classrooms were videotaped for three mathematics lessons. Utilizing a total of the resulting 171 valid video records, Q. Li and Ni (2011) analyzed characteristics of the instructional tasks in the classes along three dimensions: the level of cognitive demand of instructional tasks, types of representations involved in the tasks, and number of tasks for which teachers pursued multiple solution methods. Features of teacher–student discourse were examined along the following dimensions: cognitive levels of teacher questions, cognitive levels of student answers, types of teacher responses to student answers, and kinds of classroom authority in evaluation of student answers.

Q. Li and Ni (2011) showed that the curriculum reform achieved some of its desired goals. The reform classrooms used a significantly higher proportion of instructional tasks with high cognitive demand, in comparison to the teachers using the conventional curriculum. In addition, students in the reform classrooms were given a higher proportion of mathematical tasks that when engaged explored multiple solution methods and used multiple representations. Reform teachers were less likely to ask questions requiring recall of known answers but more likely than the non-reform teachers to ask questions that require students to describe a procedure leading to an answer. In addition, students in the reform classrooms were more likely to evaluate each other’s responses or to pose questions. However, regardless of whether teachers were in the reform or non-reform classrooms, a majority of the teachers simply repeated students’ answers as a way of showing their approval. Students from both the reform and non-reform classrooms seldom raised a question or expressed their disagreement in classroom discussion.

These results indicate that the changes in classroom instruction related to instructional tasks were more evident than for patterns of classroom discourse. These results suggest that the empirical question of the relations of instructional tasks to classroom discourse should be examined more thoroughly. Therefore, the objective of the present study, utilizing the data from Q. Li and Ni’s study (2011), was to examine the relations between instructional tasks and the nature of classroom discourse.
THEORETICAL BACKGROUND AND PURPOSE OF THE STUDY

Instructional tasks and classroom discourse, key factors influencing students’ experience and learning in classroom, are seen as separate but simultaneous entities woven together in classroom instruction (Hiebert & Wearne, 1993; Stein et al., 1996). While they are two distinctive theoretical constructs, they interact in classroom settings, and this latter empirical relationship needs to be more explicitly investigated. In the following sections, relevant literature will be drawn upon to explain the two constructs and their relationship.

Instructional Tasks

An instructional task refers to a classroom activity or a segment of classroom work that intends to focus the attention of students on particular domain knowledge or skills (Doyle, 1983). For example, a Chinese teacher used the instructional task shown in Figure 1 to teach her fifth-grade students how to calculate the volume of rectangular prisms.

Academic tasks are the basic unit of classroom instruction. A collection of academic tasks constitutes curriculum requirements. To illustrate the differences between types of instructional tasks, Doyle (1983, 1988) classifies instructional tasks in various school subjects into three major classes: memory tasks, procedural or routine tasks, and comprehension or understanding tasks.

Instruction: Each group has 40 cubes with sides of 1 cm, please design different rectangular prisms using the small cubes and complete the form below.

<table>
<thead>
<tr>
<th></th>
<th>Length /cm</th>
<th>Width /cm</th>
<th>Height /cm</th>
<th>Number of cubes</th>
<th>Volume /cm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prism 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prism 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prism 3</td>
<td></td>
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<td>......</td>
<td></td>
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</tr>
</tbody>
</table>

(1) Observe the data in the above form, what do you find?

(2) How can you get the volume of a rectangular prism?

FIGURE 1 Fifth-grade instructional task on prisms.
Literature has indicated that the nature of specific learning tasks can influence learners by directing their attention to particular aspects of content and by specifying ways of processing information (Anderson, 2000; Doyle, 1983; Hiebert & Wearne, 1993; Ni & Zhou, 2005).

By building on Doyle’s framework (1983, 1988) as well as the ideas of mathematical reform curricula in the United States (National Council of Teachers of Mathematics, 1989, 1997), Stein and her colleagues (Henningsen & Stein, 1997; Stein et al., 1996, 2000; Stein, Remillard, & Smith, 2007) developed an instructional task framework to explore how instructional tasks influence student learning. The framework consisted of three significant components. The first is that instructional tasks can be represented at three connected levels, primarily as they appear in curricular and instructional materials, then as they are engaged by classroom teachers and finally, as they are actually experienced by students. The influence of instructional tasks on student learning is realized through the chain of task implementation. Second, there can be differences in the cognitive demand of an instructional task, changing from its original inception to eventual implementation. The characteristics of an instructional task can be transformed in a classroom by various factors other than the task itself (Doyle, 1983, 1988; Henningsen & Stein, 1997; Stein, Grover, et al., 1996).

In the final component, Stein et al. (1996, 2000) designed a coding system to characterize cognitive features of a mathematics instructional task. The system itself contains three dimensions: cognitive demand of an instructional task, single or multiple representations involved in the task, and single or multiple solution methods explored towards the task. The researchers further differentiated four levels of cognitive demand of mathematics tasks: as memorization, as procedures without connecting them to underlying concepts and meanings, as procedures with the connections, and doing mathematics. Memorization tasks involve either reproducing or committing to memory previously learned facts, rules, formulae, or definitions. Procedures without connections are algorithmic, and they have little connection to the concepts or meaning that underlie the procedures being used. Procedures with connections are tasks that focus students’ attention on the use of procedures for the purpose of developing mathematical concepts. Tasks characterized as doing mathematics require students to explore and understand mathematical concepts, processes, or relationships by using complex, nonalgorithmic thinking.

These three cognitive dimensions of mathematics instructional tasks are shown to influence student learning in different grade levels (Hiebert & Wearne, 1993; Stein & Lane, 1996; Stigler & Hiebert, 2004). This line of research has provided the impetus for the current mathematics curriculum reform in many countries including China.

Classroom Discourse

Classroom discourse refers to the conversation that takes place between the teacher and his or her students and among the students in classroom instruction while a class is officially in session. However, it should be noted that the investigation of classroom discourse in the present study focused on the teacher–student conversations that concern the content of the subject being studied. In classroom discourse, the teacher is responsible for controlling the direction of the dialogue. Meanwhile, the teacher and the students in the interaction become environments for each other (Fairclough, 1992; Hicks, 1995). In addition, the particular instructional tasks become part of the context of interaction. Vygotsky’s theory of the social formation of mind that emphasizes the
critical role of social interactions with language in shaping higher mental functions (Vygotksy, 1997) has provided the direction for Cazden (2001) to synthesize the nature of classroom discourse. Cazden described the process of classroom discourse as uniting three functions of classroom life: “the communication of propositional information, the establishment and maintenance of social relationships, and the expression of the speaker’s identity and attitudes” (p. 3). In analyzing the functions of teacher questions to shape classroom discourse, Williams and Baxter (1996) view teacher questions as serving to build social scaffolding to help establish expectations for participation and authority for classroom discourse or to build analytical scaffolding to support students as they process information to construct knowledge.

As indicated in the previous section, the new Chinese mathematics curriculum encourages teachers to pay more attention to the processes in which knowledge is constructed and advanced. This necessarily entails changes from what were previously one-way classroom dialogues (see Figure 2) to more interactive and open classroom discourse where conversation between students and teachers varies more (see Figure 3). Students are expected to assume greater learning responsibility, to become more involved in discussions and share ideas on learning, to make conjectures and to evaluate evidence.
Relations Between Instructional Tasks and Classroom Discourse

For the purpose of this article, instructional tasks and classroom discourse are regarded as two different entities as described above. This distinction is necessary in order to investigate the relationship between them. Before providing an explanation for their association, a brief discussion helps to clarify the way in which the two constructs are different in terms of their respective roles in curriculum as well as the extent to which they may be regulated by designated curricula in the Chinese cultural context.

As indicated earlier, China has a nationally mandated school mathematics curriculum. As a result, the development and publishing of textbooks is closely regulated and monitored by the central government, the Ministry of Education. Only a few officially designated publishers are allowed to develop textbooks and teacher manuals for public schools with the approval of a central government-appointed committee. Usually it is a city-level school district that decides which textbook series will be used in schools within the district. The relatively strict and normative system governing school curriculum might contribute to the findings that the impact of curriculum materials on what teachers actually teach appeared to be greater in China than in other countries (Cai, 2005; Han & Paine, 2010; X. Li, Ding, Capraro, & Capraro, 2008). However, in contrast to a rapid shift in the types of instructional tasks appearing in China’s elementary mathematics classrooms, significant change in classroom discourse is a slow process (Q. Li & Ni, 2011), in part because the context of Chinese classrooms with well-developed cultural norms creates a complex situation for the change to occur.

The discussed characteristics of instructional tasks and classroom discourse appear to complement each other as they interweave constantly in teaching and learning. On one hand, the particular content, physical features, and cognitive demands of instructional tasks provide an environment that may facilitate certain kinds of classroom discourse. For example, an instructional task that affords multiple solution methods would be more likely to invite student responses. On the other hand, the way that the teacher and students discuss a given instructional task may either “amplify” or limit the intended utility of the task. Consequently, whether or not the teacher’s question requires making connections with the key concepts or principles intended by a task would affect students’ learning opportunities.

The fundamental question remains as to how instructional tasks and classroom discourse constrain and interact with each other in the actual classroom setting (Hiebert & Wearne, 1993; Remillard, 2005). Stein’s work (Henningsen & Stein, 1997; Stein et al., 1996, 2000) examined how the cognitive demands of mathematics instructional tasks might change from the way that teachers set them up to the way students actually experienced them. They found that teachers tended to reduce the level of cognitive demand when they enacted tasks of high cognitive demand. For example, over half of the tasks that were set up to require the use of procedures with meaningful connections failed to keep the connection to meaning in their classroom implementation.

Stein et al. (1996; Henningsen & Stein, 1997) identified several factors that may compromise the cognitive demand of tasks. The factors include when the problematic aspects of a task become routine, the tendency of the teachers to do the challenging aspects of the task for the students, and the tendency of the teachers to shift the focus from solution processes, meaning, concepts, and understanding to the correctness or completeness of the answer. In contrast, the following factors appeared to be beneficial for sustaining the cognitive vigor of learning tasks. They include when the teacher or capable students model competent performance, when the teacher provides
necessary support but does not assume control of student work, and feedback consistently given to students to encourage justification and explanation of their answers.

Related to the classroom factors that may facilitate or prohibit the realization of the potential utilities of high cognitive demand tasks, Baxter and Williams (2010) and Williams and Baxter (1996) found that the American middle school mathematics teachers were very effective in providing social scaffolding to increase student talk but were reluctant to provide analytical scaffolding. The observed teachers encouraged the students to look for alternative strategies and valued multiple approaches to problems, consistent with the discourse-oriented approach promoted by the reformed mathematics curricula. However, the teachers faced a dilemma of finding a balance between promoting social interactions and ensuring that students gain specific mathematics skills and knowledge. These studies illustrate how the nature of classroom discourse might sustain or restrain the utility of instructional tasks.

Characterizing Multiple Solution Methods With Respect to Tasks and Discourse

It is necessary to explain the issue concerning the nature of the variable of multiple solution methods in the present study. The variable has been considered a task variable in the existing literature (Stein et al., 1996, 2000), although Stein and her colleagues did not provide a concrete example of how they coded this variable in their studies. The Chinese mathematics curriculum standards (Ministry of Education, 2001b) and the accompanying teacher manual for fifth-grade mathematics (Working Group for School Mathematics of Nine-Year Compulsory Education, 2001) state explicitly that the emphasis of problem-solving approaches in mathematics instruction promotes the use of the instructional tasks that permit multiple solution methods and the utilization of multiple perspectives from students. In our data, of the 525 tasks identified in the observed 90 class sessions, the teachers pursued multiple solution methods for 200 of them (38.1%). The percentage suggested that the curriculum materials influenced teachers’ selection and use of tasks to encourage multiple solution methods. Therefore, we initially treated multiple solution methods as a task variable.

However, further inspection of the data revealed cases where multiple solution methods were pursued largely due to the decisions of the teacher after a student produced an initial solution method. In the project classrooms, multiple solution methods appeared in two ways. Either the teacher gave an explicit oral requirement to the class to produce more than one solution, or the teacher displayed the feature of multiple solution methods over the course of the discussion of the solution. In the majority of the cases, teachers did not make explicit and upfront oral requirements for multiple solution methods.

To balance the considerations discussed above, we decided to treat multiple solution methods as a variable of both the task and the discourse in the Chinese classrooms. We labeled this variable as Teacher’s Pursuit of Multiple Solution Methods. We acknowledge that this might not be a clean way to treat the variable, but we feel it is an intellectually honest way based on our understanding of the curriculum materials and the classrooms.

Purpose of the Study

The existing literature has provided valuable insight into the dynamics between instructional tasks and classroom discourse. However, the previous studies (Henningsen & Stein, 1997; Y. P. Ma,
2005; Stein et al., 1996; Yu, 2003) have not related specific features of tasks to specific features of classroom discourse. More studies are needed in order to understand the particular details of the way the two affect each other in the classroom setting. Understandably, given that teaching and learning is highly culturally bound, it will be also worthwhile to investigate the issues in a cultural setting such as Mainland China, one that is different from those that have dominated the existing literature and has well-developed cultural norms for teaching and learning mathematics (Fan et al., 2004). Therefore, the purpose of the present study was to examine and describe the relationships among features of instructional tasks and classroom discourses in Chinese classrooms that were implementing the current reforms.

In the study, instructional tasks were considered as an instructional condition prior to classroom discourse (Doyle, 1983; Stein et al., 2007). The researchers then examined the way that instructional tasks were related to elements of classroom discourse and how instructional tasks with the antecedent elements (e.g., teachers’ questions) of classroom discourse affected later elements (e.g., students’ answers). Specifically, the study investigated the following hypotheses based on the existing literature:

1. Tasks with high cognitive demand or with multiple representations make it likely that teachers would guide students to pursue multiple solution methods towards the tasks.
2. Tasks with high cognitive demand or multiple representations or for which multiple solutions are pursued make it likely that teachers would ask higher order questions (i.e., explanatory questions or analytical questions).
3. Tasks with high cognitive demand or multiple representations, for which multiple solution methods are pursued, or teachers’ higher order questions make it likely that students will provide highly participatory answers (i.e., share their reasoning, comment on others’ answers, or raise new questions).
4. Tasks with high cognitive demand or multiple representations, for which multiple solution methods are pursued, or higher order teacher questions, or highly participatory student responses make it likely that the teacher would inquire further into students’ responses.
5. Tasks with high cognitive demand or multiple representations, for which multiple solution methods are pursued, or higher order teacher questions, or highly participatory student responses, or teachers’ exploratory responses make it likely that teacher–student joint authority in evaluating students’ responses will occur in the classrooms.

METHOD

Data Source

The present study analyzed the data from the Q. Li and Ni (2011) study, which involved 60 fifth-grade mathematics teachers and their classrooms from 20 schools. Half of the classrooms randomly selected from one school district had adopted the new curriculum, and the other half from another district used the conventional curriculum. The present study utilized the data involving only the 30 reform classrooms and their 1,779 fifth-grade students (990 male students and 789 female students), because only those classrooms were using the new curriculum and teaching practices required by the reform.
The students in the 30 classes had been taught with the reformed curriculum since their first grade. The students were assigned to the schools according to the schools’ proximity to where they lived. The profiles of the teachers were representative of all of the elementary mathematics teachers in that city. Among the 30 teachers, 22% of them had a bachelor’s degree and the remaining had either an associate’s degree or a normal college degree (from a college offering only teacher training programs of two or three years). Nineteen percent of them had 1–5 years of teaching experience, 25% with 6–10 years of teaching experience, 31% with 11–15 years of experience, and 25% with more than 16 years of experience. The average class size was 59 students. The teachers had engaged with the reform curriculum for different numbers of years of experience, ranging from 1–5 years; most had 1–3 years of experience with the new curriculum when the study took place. Teachers were assigned to classrooms at different times as teaching assignments were usually arranged by grade-block in the schools. One group of teachers would teach grades 1 and 2 mathematics, another group would teach grades 3 and 4, and another group would teach grades 5 and 6.

Each of the 30 teachers and their classrooms was observed for three consecutive days, and all the 90 total observations were completed within one month in November to December of 2006. Each lesson lasted for about 45 minutes. The teachers were informed of their respective observation schedules one week before the observation took place. Teachers were also informed that the goal of the study was to see what would typically happen in Chinese mathematics classrooms using the reformed curriculum. Therefore, they were instructed to teach according to the curriculum schedule (which specified what to teach) in the usual way that they conducted classroom teaching and with no extra preparation. The teachers’ written lesson plans for the videotaped lessons were also collected. Eighty-one out of the 90 lessons introduced new knowledge, and the remaining nine were review or exercise lessons. The content presented in the videotaped lessons covered the least common multiple, common denominators, and irreducible fractions (nine lesson sessions of three classes), reciprocals (11 lesson sessions of 11 classes), division with fractions and equations with one unknown to solve word problems involving fractions (37 lessons of 16 classes), and shapes and their measurements (33 lesson sessions of 11 classes).

Coding the Lesson Data

Initially, the videotaped class sessions were transcribed in texts. They were subsequently coded in terms of the features of instructional tasks and the features of classroom discourse. The coding procedures to analyze the features of instructional tasks and classroom discourse were described in detail in Q. Li and Ni (2011). However, for the convenience of a reader, we briefly explain the procedures here.

**Identifying Instructional Tasks.** A mathematical task was defined as a classroom activity or a segment of classroom work, the intention of which is to focus the attention of students on a particular mathematical idea. Any self-contained mathematics problem, activity or exercise that involved the students during a lesson was counted as a math task. In most cases, a task implemented in class was a single problem or activity, such as the following:
Ming Ming wants to make a colorful box with Length = 12 cm, Breadth = 10 cm, and Height = 8 cm as a present to give his sister. What will be the minimum surface area of the gift paper that will completely wrap the box?

In some other cases, however, a task might consist of a set of similar questions implemented during the same segment of a classroom work. For example, a teacher could present a task on the reciprocals of the numbers, “What are the reciprocals of these numbers respectively: 1/3, 3/4, 1, 8/7, 2, 5/6, 3?” Since the students solved these similar problems with the same or relevant knowledge, the problems were presented in the same classroom segment, and the problems were all focused on reciprocals, the set of similar problems was counted as one task. To establish the interrater agreement on the number of tasks for a lesson, 14% or 24 of the 171 videotaped lessons, including 14 from the reform group and 10 from the non-reform group, from the 60 classrooms were randomly selected for double coding (Q. Li & Ni, 2011). First, all the problems or activities were taken out of the transcriptions of the lessons according to the identified teaching segments. The raters (experienced elementary mathematics teachers and graduate students in mathematics education) then identified individual tasks according to the considerations described above. In addition, they referred to the teachers’ written lesson plans. Using this procedure, interrater agreement for identifying the number of instructional tasks in the lessons ranged from 0.84–0.93 (Q. Li & Ni, 2011). The identified instructional tasks from the 24 lessons then were doubled coded along the three task features as described below.

**Characterizing Instructional Tasks.** The identified instructional tasks included those written on the blackboard, or presented by the teachers orally, or taken from the student textbooks as long as they were actually used for the instruction in the observed class sessions. They were coded along the dimensions of (a) the level of cognitive demand required to solve the task and (b) the extent to which the task involves multiple representations (Renkl and Helmke, 1992; Stein et al., 1996, 2000; Stigler and Hiebert, 1999).

Instructional tasks were first classified into two major types, namely tasks of lower level cognitive demand and those of higher level cognitive demand (Doyle, 1983; Stein et al., 1996). The former includes memorization tasks and tasks of procedure without connection; the latter refers to the tasks that make connections between procedures and underlying concepts and the tasks of doing mathematics. Majority of the tasks under the group of high cognitive demand were those with procedures with connections in the study. Cohen’s Kappas coefficient for tasks of high cognitive demand was 0.87.

An identified instructional task was coded as whether it employed a single representation or multiple representations. When a teacher not only used the arithmetical form (e.g., \( \frac{6}{7} + \frac{1}{3} = ? \)), but also a graphic form, or other visual illustrations, or manipulatives to represent an instructional task, that task was coded as one of multiple representations. Three kinds of presentation were coded, including numerical symbols, graphic illustrations (e.g., a figure or a table), and physical representations that students could manipulate by

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1The current study only analyzed 90 lessons out of the 171 videotaped lessons that were collected from the 60 classrooms. The 90 lessons were from 30 of the classrooms that were implementing the new mathematics curriculum.
hand (e.g., Lego bricks). Cohen’s Kappas coefficient for tasks of multiple representations was 0.90.

**Coding Teachers’ Pursuing Multiple Solution Methods.** A clarification of the term is needed here. The original term was *multiple solution strategies* from Stein et al. (Henningsen & Stein, 1997; Stein & Lane, 1996; Stein et al., 2000). Some mathematics tasks can have just one solution but multiple methods to arrive at the solution. For example, the task “Which fraction is the largest and which is the smallest: one third, one fifths, and 3/25?” has only one solution but the solution can be reached with different methods. Some other tasks can have more than one solution as well as multiple methods to lead to them. The following task is such an example:

Both Jane and Mary take a summer job. Jane earns $30 a day whereas Mary earns $20 a day. How many days do Jane and Mary have to work respectively in order for the two to earn the same amount of money?

The term *multiple solution strategies* by Stein et al. (2000) is supposed to apply to both types. However, in the present study, almost all instructional tasks observed in the fifth-grade classrooms were of the first type. Therefore, we use the term *multiple solution methods* to refer to the tasks that can be solved in multiple ways but have a single response.

As mentioned before, there were two ways showing the feature of multiple solution methods in the classrooms. In one, a teacher gave an explicit oral requirement for multiple solution methods upfront. For example, a teacher wrote the question on the blackboard “Find out the least common multiple for number 24 and 36.” Then, she announced immediately “There can be different ways to find the least common multiple of the two numbers, and please use your own method to do it.” The teacher then solicited different methods one after another from the students in the following example:

Classroom Episode 1

(The teacher is writing the two numbers, 24 and 36, on the blackboard. The teacher asks one student to do the task on the blackboard, and the rest of the class is doing the task at their desks.)

1. Teacher: What did you get for the least common multiple for the two numbers? Ting (the student’s name)
2. Student 1: 72.
3. Teacher: What method did you use to get it?
4. Student 1: I used the short division. 2 is the common divisor to obtain the quotient 12 and 18. To find the common divisor 3 for 12 and 18, I then got the quotient 4 and 6. Find the common divisor again and I got the quotient 2 and 3. Finally, I have the common divisors, 2, 3, 2 and the quotients 2 and 3 multiplied and the answer is 72.
10. Teacher: Could you come up with a different method to get the least common multiple?
11. Student 2: I used the method of listing, 24, 48, 72, 114; then 36, 72, 114, the least common multiple is 72.
12. Teacher: Good.

In the identified 200 tasks in which teachers pursued multiple solution methods, only six of those in five class sessions involved the teachers’ explicit and upfront oral requirements for multiple solution methods. The small number prompted our decision to treat the variable as it contains both the aspect of task and that of discourse. However, the six tasks were not removed from the data because they contained the parts in which the teachers asked for different methods after the students provided an initial method as illustrated in the above classroom example.

It should be noted that the variable was coded in terms of number of tasks in which a teacher pursued multiple solution methods with his or her students but not in terms of how frequently they pursued multiple solution methods for each given task. Cohen’s Kappas coefficient for the tasks for which teachers pursued multiple solution methods was 0.93 (Q. Li & Ni, 2011).

**Coding the Classroom Discourse.** We coded the classroom discourse by focusing on questions from teachers, answers from students, the teachers’ reactions to the responses of students, and the nature of authority, teacher authority or teacher–student joint authority for evaluating a student’s response (e.g., Cobb, Wood, Yackel, & McNeal, 1992; Hamm & Perry 2002; Q. Li, 2004; Stigler, Fernandez, & Yoshida, 1996; Stigler & Hiebert, 1999). The randomly selected 24 videotaped lessons were double coded as well along the dimensions of classroom discourse.

**Teachers’ Questions: Lower Order and Higher Order Questions.** The coding of teachers’ questions draws on the ways in which teachers use questions to stimulate students’ specific mathematical thinking (Perry, VanderStoep, & Yu, 1993). Four types of teacher questions were observed:

1. **Memory recall and confirmation questions** require students to repeat facts, procedures or mathematical rules that are previously taught in the class. For example, “What is the reciprocal of number 2?” Confirmation questions are used to confirm with students whether or not they used the same procedure or obtained the same answer. For example, “Have you got the same answer as his?”
2. **Procedural and descriptive questions** ask students to describe the procedure that leads to an answer, for example, “How do you get this least common multiple?” Descriptive questions ask students to describe the literal meaning of a word problem. For example, “What does the sentence ‘two fifths of the 29 children are girls’ mean?”
3. **Explanatory questions** require students to describe their thinking in selecting strategies and procedures for solving problems or to describe why a certain procedure works. For example, “Why do you group the questions in the two types?”
4. Analytic and comparative questions ask students to consider the nature of a problem or of a certain strategy or to compare two students’ thought processes and consider connections between them. For example, “Which of the two methods to find the least common multiple is more efficient to solve this particular problem?”

According to Hiebert and Wearne (1993), explanatory questions and analytic questions are the types of questions that prompt students to “examine the underlying features” of a problem (p. 42). For the present study, these questions were coded as Higher Order Questions, whereas memory recall questions and questions that ask for describing procedures or the literal meaning of a word problem were coded as Lower Order Questions.

Students’ Answers: Simple and Highly Participatory Answers. Five kinds of student answers were coded:

1. Simple answers provide just “Yes” or “No” answers.
2. Descriptive answers describe a procedure taken to produce an answer or the meaning of a sentence in a word problem. For example, in answering a teacher’s question, “How did you get 4/7 ÷ 2 = 2/7?” a student responds, “Divide a piece of paper into seven parts equally, get the four parts, then divide the four parts into two and get 2/7.”
3. Explanatory answers explain why a certain strategy is applied. For example, when asked, “Could you explain why 0 does not have a reciprocal?” a student answered, “I think that 0 has no reciprocal because 0/0 is not logical.”
4. Commentary answers evaluate different solution approaches or comment on other student answers. For example, “I disagree with X because 0 can not be used as a denominator.”
5. A student raising a new question provides new opportunities to explore further of a given topic, such as “Why does 4/7 ÷ 2 become 4/7 × 1/2?”

For the present study, simple answering and descriptive answering were grouped as Simple Answers. Students’ explanatory answering, commentary answering and raising new questions were regarded as Highly Participatory Answers because they reflected students’ independence in their mathematical reasoning and contributions to the classroom discourse.

Teachers’ Exploratory Responses. The coding category of teachers’ response was concerned with how the teachers reacted to students’ answers. According to Hamm and Perry (2002), a teacher may have four different reactions to a student’s answer:

1. The teacher abandons or ignores the student’s idea. For example:
   1) Teacher: Which is bigger, one half or one third?
   2) Student: One third.
   3) Teacher: Not really (and continues teaching).
2. The teacher acknowledges the student’s reply but does not incorporate it into the lesson. For example, after a student answers one question, the teacher says, “Very good,” and then continues teaching.
3. The teacher repeats the student’s idea to express approval. For example:
   1) Teacher: How do you know the answer is 19?
   2) Student: Because it has nine 1’s and one 10.
3) Teacher: Yes, one 10 and nine 1’s. (and then changes the subject and continues teaching).
4) The teacher examines and utilizes a student’s idea and clarifies it. This is especially useful when a student has a novel idea or has made a mistake. For example, after a student produces the answer that one seventh is bigger than one fifth, a teacher asks, “Can you draw to prove your answer?”

In this study, the fourth type of teacher explorative response was of interest, as it can initiate further class discussion by utilizing or exploring a student’s idea. We also assumed that this type of teacher explorative response was positively related to the use of instructional tasks with high cognitive demand, multiple representations or multiple solution methods.

**Evaluation Authority: Teacher Alone and Teacher and Student Jointly.** The coding category of the evaluation authority designates the source of authority in classroom discourse (Hamm & Perry, 2002). Teacher authority was present when correctness was assessed by the teacher alone through verbal or nonverbal reaction to students’ answers. For example: A student provides the answer “11” to the question “5 + 5 + 1=?” and the teacher responds, “Correct!” In contrast, teacher–student joint authority indicates that the teacher and students worked together to establish correctness. For example, a teacher invites students to evaluate a peer’s answer “Does everyone agree?” or “Let’s do the problem together to see if his answer is right.” Textbook authority means that the textbook established the correctness. For example, “Let’s take a look at the example in the textbook to see how the problem was done.” Very few examples of textbook authority occurred in the classrooms. Therefore, textbook authority was not considered in further analysis.

Cohen’s Kappas coefficients for interrater agreement were 0.780 for teacher questions, 0.763 for student answers, 0.756 for teachers’ responses to students’ answers, and 0.824 for evaluation authority.

These variables on classroom discourse were used in the regression analyses reported below. They included Teachers’ Lower Order Questions and Teachers’ Higher Order questions, Students’ Simple Answers and Students’ Highly Participatory Answers, as well as Teachers’ Exploratory Response, Teacher Authority and Teacher–Student Joint Authority.

**Statistical Analyses**

Stepwise multiple regressions were performed to test for the hypothesized relations. In particular, five sets of regression models were constructed. The first one investigated the association of the two features of instructional tasks, high cognitive demand and multiple representations, with the teachers’ pursuit of multiple solution methods. The second set examined the association of the types of teacher questions with the two features of instruction tasks, as well as with the teachers’ pursuit of multiple solution methods. The third set tested the relations of the types of student responses to the types of teacher questions, the two task features, and the teachers’ pursuit of multiple solution methods. The fourth set assessed the associations of teachers’ explorative responses with the two task properties, the teachers’ pursuit of multiple solution methods, teacher questions, and student responses. The final set inspected the associations of evaluation authority,
teacher authority or teacher–student joint authority, with the two properties of instructional tasks, the teachers’ pursuit of multiple solution methods, teacher questions, student responses, and teachers’ explorative responses.

The variables of concern were grounded at three levels. The two properties of instructional tasks, high cognitive demand and multiple representations, composed one level. The next level included the teachers’ pursuit of multiple solution methods. The final level included the elements of classroom discourse. The stepwise method was used in the last four multiple regression analyses to distinguish the contributions in predicting the other variables of the two task properties (high cognitive demand and multiple representations) and of the teachers’ pursuit of multiple solution methods. However, the stepwise method was not employed for the first regression model because the model involved only the variables of the first two levels.

The frequency scores on the variables of the 90 individual class sessions instead of the averaged scores of the 30 classrooms were used for the analyses. There were two main reasons for this choice. First, the coded responses from teachers in the same classroom varied considerably between the three videotaped lessons, especially on the variables of classroom discourse. For instance, students’ analytical answers coded in three class sessions in one classroom showed significant differences ($M = 13, SD = 11.53$). This indicates that task implementation and classroom discourse are context dependent and are influenced by lesson content and other dynamic factors. Thus, it was not appropriate to ignore the variability among individual class sessions by averaging coding scores of the three class sessions for each classroom. Second, the focus of this study was to explore the relations between high level tasks’ implementation and classroom discourse in general, not to investigate the influence of individual teachers’ or students’ attributes on their teaching or learning behaviors, respectively. Therefore, using individual class sessions rather than individual classes or teachers as the unit of analysis was most appropriate for the research.

However, using regression analyses for the 90 individual lessons violates the independence of observation assumption required for the inferential analysis if there were temporal correlations between the three observations of the same variable within the same class. Therefore, correlations between the three observations Time 1, Time 2, and Time 3 of the same 10 variables (the three task variables and the seven discourse variables) were examined. Out of 10 pairs of correlation (three task variables and seven discourse variables) of the same variable between Time 1 and Time 2 observations, eight of them ranged from .38 to .28, and the other two were .32. None of them were statistically significant. Out of 10 pairs of correlation between Time 2 and Time 3 observations, nine of them ranged from .07 to .26 and none statistically significant. The exception was that the correlation was .39 for the variable for Teacher’s Higher-Order Questions, and it was statistically significant at .05 level. For the 10 pairs of correlation between Time 1 and Time 3, seven pairs of correlation were in the range between .10–.20, and none were statistically significant. However, three pairs of correlation between Time 1 and Time 3 reached statistical significance, Highly Participatory Answers ($r = .36, p < .05$), Teacher Authority over evaluation ($r = .44, p < .05$), and Teacher–Student Joint Authority ($r = .39, p < .05$). In sum, four pairs of correlation were statistically significant out of the 30 pairs. This suggested that the lessons varied greatly within a class on the examined variables and provided a justification for our decision to use individual class sessions as the unit of analysis for the present study. Nevertheless, parallel regression analyses involving 30 classrooms were also performed with the averaged scores over three classroom observations in order to understand the extent to which the unit of analysis affected the study results.
Results obtained with the 90 lessons were reported below. The consistency of the results obtained from the sample of the 90 lessons and those from the 30 classrooms was briefly reported on as well.

RESULTS

Descriptive Results

A total of 525 tasks were identified from the 90 class sessions. Of these, 257 tasks, approximately 49% of all tasks, were considered to be Tasks of High Cognitive Demand. There were 200 tasks, approximately 38.1% of all tasks, in which teachers pursued multiple solution methods. Some tasks contained multiple subtasks. It seemed more appropriate to code the nature of representation for each subtask. Therefore, the total number of tasks was 960 that were coded on the dimension of multiple representations. Five hundred and fifty-three of them were those with multiple representations, approximately 57.6% of the 960 tasks. Of the 2,106 questions identified, 1,076 were Higher Order Questions (explanatory and analytic questions), accounting for 51% of all teacher questions. Of the 3,804 student answers counted, 1,441 of them, approximately 37.9%, were identified as Highly Participatory Answers (explanations, comments on others’ answers, or raising new questions for discussion). Of the total 1,021 teacher responses to Students’ Highly Participatory Answers, 456 teacher responses, approximately 44.7%, utilized and clarified students’ ideas in initiating further class discussion. Of 1,084 occasions of teacher evaluation of student responses, 448 instances of Teacher Authority, or 41.3%, were identified in comparison to 536 instances of Teacher–Student Joint Authority.

Table 1 provides the descriptive statistics of the variables of concern. One sample Kolmogorov-Smirnov Test was used to assess whether the variables followed a normal distribution as required by most parametric tests. The results showed that all variables were normally distributed except the variable of teacher authority (Kolmogorov-Smirnov $Z = 1.667, p < .05$). However, both its skewedness (1.162) and kurtosis (1.352) were within the range of -2 and 2. This indicates that its distribution did not violate the normal distribution dramatically and therefore could be tolerated in parametric tests with the sample size of 90.

Table 2 displays the Pearson correlations between these variables. Three sets of associations between them were noted. First, Tasks of High Cognitive Demand were positively correlated to Tasks of Multiple Representations ($r = .504, p < .001$). Tasks of High Cognitive Demands were positively associated with Teachers’ Pursuing Multiple Solution Methods ($r = .226, p < .05$). However, Tasks of Multiple Representations were not correlated to Teachers’ Pursuing Multiple Solution Methods.

Second, Tasks of High Cognitive Demand were positively correlated with Teachers’ Higher-Order Questions ($r = .217, p < .05$), Students’ Simple Answers ($r = .287, p < .05$), Teachers’ Exploratory Responses to students’ answers ($r = .410, p < .001$), and Teachers’ Authority in evaluation of student’s answers ($r = .492, p < .001$). Tasks of Multiple Representations were

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For a task that contained several subtasks, each subtask was coded on the nature of task representation. This variable was coded this way in order to increase its variability between the classes. Therefore, the total number of tasks along the dimension of multiple representations was larger than that on the dimension of cognitive demand and multiple solution methods (see Table 1).
## TABLE 1
Means, Standard Deviations, 25% and 75% Quartile of Task Variables, Teachers’ Pursuing Multiple Solution Methods, and Classroom Discourse Variables (n = 90)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>25%</th>
<th>75%</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tasks</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tasks of high cognitive demand</td>
<td>0</td>
<td>11</td>
<td>2.860</td>
<td>1.75</td>
<td>4</td>
<td>2.096</td>
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<tr>
<td>Tasks of multiple representations</td>
<td>0</td>
<td>17</td>
<td>6.144</td>
<td>3</td>
<td>8</td>
<td>4.041</td>
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<tr>
<td>Teachers’ Pursuit of Multiple Solution Methods</td>
<td>0</td>
<td>8</td>
<td>2.222</td>
<td>1</td>
<td>3</td>
<td>1.634</td>
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<td>Classroom Discourse: Teachers’ questions</td>
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<td></td>
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<td>Lower order questions</td>
<td>0</td>
<td>40</td>
<td>11.444</td>
<td>6</td>
<td>16</td>
<td>7.482</td>
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<td>Higher order questions</td>
<td>0</td>
<td>36</td>
<td>11.956</td>
<td>8</td>
<td>15</td>
<td>6.148</td>
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<tr>
<td>Classroom Discourse: Student’s answers</td>
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<tr>
<td>Simple answers</td>
<td>0</td>
<td>62</td>
<td>26.256</td>
<td>18.75</td>
<td>32</td>
<td>12.595</td>
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<tr>
<td>Highly participatory answers</td>
<td>0</td>
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<td>16.011</td>
<td>9</td>
<td>21.25</td>
<td>9.019</td>
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<td>Classroom Discourse: Teacher’s response</td>
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<tr>
<td>Exploratory responses</td>
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<td>5.067</td>
<td>1</td>
<td>8</td>
<td>4.357</td>
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<td>Classroom Discourse: Evaluation authority</td>
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<tr>
<td>Teacher authority</td>
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<td>4.978</td>
<td>2</td>
<td>8</td>
<td>4.409</td>
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<tr>
<td>Teacher–student joint authority</td>
<td>0</td>
<td>25</td>
<td>6.056</td>
<td>2</td>
<td>8.25</td>
<td>4.719</td>
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## TABLE 2
Correlations between the Task Variables, Teachers’ Pursuing Multiple Solution Methods, and Classroom Discourse Variables (n = 90)

<table>
<thead>
<tr>
<th>Variables</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Tasks of high level cognitive demand</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2. Tasks of multiple representations</td>
<td>.504***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Teachers’ pursuing multiple solution methods</td>
<td>.226*</td>
<td>.087</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>4. Teachers’ lower order questions</td>
<td>.198</td>
<td>.126</td>
<td>.301**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Teachers’ higher order questions</td>
<td>.217*</td>
<td>.154</td>
<td>-.191</td>
<td>.117</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>6. Students’ simple answers</td>
<td>.287*</td>
<td>.113</td>
<td>.313**</td>
<td>.530**</td>
<td>.124</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>7. Students’ highly participatory answers</td>
<td>-.020</td>
<td>-.048</td>
<td>-.056</td>
<td>.065</td>
<td>.412***</td>
<td>-.083</td>
<td></td>
<td></td>
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<tr>
<td>8. Teachers’ exploratory response</td>
<td>.410***</td>
<td>.430***</td>
<td>-.092</td>
<td>.146</td>
<td>.383***</td>
<td>.126</td>
<td>.320**</td>
<td></td>
<td></td>
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<tr>
<td>9. Teacher authority in evaluation</td>
<td>.492***</td>
<td>.334**</td>
<td>.049</td>
<td>.192</td>
<td>.482**</td>
<td>.330**</td>
<td>.017</td>
<td>.508***</td>
<td></td>
</tr>
<tr>
<td>10. Teacher–student joint authority</td>
<td>.195</td>
<td>.218*</td>
<td>.071</td>
<td>.377***</td>
<td>.208*</td>
<td>.225*</td>
<td>.361***</td>
<td>.505***</td>
<td>.089</td>
</tr>
</tbody>
</table>

*Note. The correlations were calculated based on the Z scores of the variables.  
*p < .05, 2-tailed.  **p < .01, 2-tailed.  ***p < .001, 2-tailed.*
positively related to Teachers’ Exploratory Responses \( (r = .430, p < .001) \), Teacher Authority \( (r = .334, p < .01) \) as well as with Teacher–Student Joint Authority \( (r = .218, p < .05) \). Teachers’ Pursuing Multiple Solution Methods was positively correlated with Teachers’ Lower Order Questions \( (r = .301, p < .01) \) and with Students’ Simple Answers \( (r = .313, p < .01) \).

Third, as to the associations among the components of classroom discourse, Teachers’ Lower Order Questions were positively correlated with Students’ Simple Answers \( (r = .530, p < .001) \) and Teacher-Student Joint Authority in evaluation of student’s answers \( (r = .377, p < .001) \). In contrast, Teachers’ Higher Order Questions were positively associated with Students’ Highly Participatory Answers \( (r = .412, p < .001) \), Teachers’ Exploratory Responses \( (r = .383, p < .001) \), Teacher Authority in evaluation \( (r = .482, p < .001) \), and Teacher-Student Joint Authority \( (r = .208, p < .05) \). Students’ Simple Answers were positively related to Teacher Authority \( (r = .330, p < .01) \) as well as to Teacher-Student Joint Authority \( (r = .225, p < .05) \). Students’ Highly Participatory Answers were positively associated with Teachers’ Exploratory Responses \( (r = .320, p < .01) \) and with Teacher–Student Joint Authority in evaluation \( (r = .361, p < .001) \). Teachers’ Exploratory Responses were positively correlated with Teacher Authority \( (r = .508, p < .001) \) as well as with Teacher–Student Joint Authority in evaluation \( (r = .505, p < .001) \).

Some of the above correlations are easier to interpret than others (e.g., a positive association of High Cognitive Demand Tasks with Teachers’ Higher Order Questions as compared to a significant association of Teachers’ Pursuing Multiple Solution Methods with Teachers’ Lower Order Questions). The relationships will be clarified to some degree in the regression analyses that follow.

Results of Multiple Regression Analyses

The multiple regression analyses were then conducted to test the relations between the two properties of instructional tasks, the teachers’ pursuit of multiple solution methods, and the elements of classroom discourse. We decided to use class episodes to help illustrate the phenomenon under investigation in the Chinese classrooms. We chose those classroom episodes to provide specific portraits of teacher–student interactions in the observed Chinese classrooms.

**Predicting Teachers’ Pursuing Multiple Solution Methods.** As we have indicated, Teachers’ Pursuing Multiple Solution Methods was most often deployed during the discussion of the solution to a problem. A regression model examined whether the two properties of instructional tasks, high cognitive demand and multiple representations, could predict it. As shown in Table 3, the total R-square change for the regression model of Teachers’ Pursuing Multiple Solution Methods was 5.2%, and the regression model was not significant in explaining the variance change in the variable \( (ΔF_{(2, 87)} = 2.388, p > .05) \). Nevertheless, the regression coefficient of Tasks of High Cognitive Demand was significant in predicting Teachers’ Pursuing Multiple Solution Methods \( (β = .244, p = .046) \).

**Predicting Teachers’ Higher Order and Lower Order Questions.** To predict Teachers’ Lower Order Questions and Higher Order Questions, two stepwise multiple regression models were established with the task variables, Tasks of High Cognitive Demand, and Tasks of Multiple
Representations as the predictors entered at the first step and Teachers’ Pursuing Multiple Solution Methods toward the tasks at the second step. The results are presented in Table 4.

The regression model of teachers’ Lower Order Questions was not significant at the first step. This suggested that Tasks of High Cognitive Demand, and Tasks of Multiple Representations do not predict Teachers’ Lower Order Questions. With the entrance of Teachers’ Pursuing Multiple Solution Methods at the second step, the total R-square change for the regression model of teachers’ Lower Order Questions reached 7%, a significant change with $\Delta F(1, 86) = 6.720$, $p < .05$. Teachers’ Pursuing Multiple Solution Methods was significant in positively predicting teachers’ Lower Order Questions ($\beta = .271$, $p < .05$).

The regression model for predicting teachers’ Higher Order Questions was not significant either at the first step. However, the entrance Teachers’ Pursuing Multiple Solution Methods at the second step led the total R-square change for the regression model of Teachers’ Higher Order Questions reached 6%, a significant change, $\Delta F(1, 86) = 5.809$, $p < .05$. The regression coefficient was positively significant for Tasks of High Cognitive Demand ($\beta = .248$, $p < .05$) but negatively significant for Teachers’ Pursuing Multiple Solution Methods ($\beta = -.252$, $p < .05$). These results indicated that the teachers tended to raise lower order questions when they pursued multiple solution methods for the tasks. In contrast, they were more likely to pose higher order questions in utilizing tasks of high cognitive demand. Also, the inclusion of the predictor Teachers’ Pursuing Multiple Solution Methods in the multiple regression model led to the regression coefficient for the predictor Tasks of High Cognitive Demand in association with Teachers’ Higher Order Questions to be larger than it was in the absence of the Teachers’ Pursuing Multiple Solution Methods variable.4

In a previous study (Q. Li & Ni, 2011), Chinese teachers used more higher order questioning in the reform classrooms where tasks of high cognitive demand were encouraged compared to those using the conventional curriculum. In viewing our transcripts, we found that 54 of 90 lessons used more than 10 higher order questions to ask students to explain the process to solve a given problem. Classroom Episode 2 below shows an example of classroom discourse between teacher

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**TABLE 3**

Multiple Regression Analysis of Predicting Teachers’ Pursuing Multiple Solution Methods From the Task Variables

<table>
<thead>
<tr>
<th>Predictors</th>
<th>$R^2$ change ($\Delta R^2$) and $F$ change ($\Delta F$)</th>
<th>Std. Error</th>
<th>Beta</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tasks of high cognitive demand</td>
<td>$\Delta R^2 = .052$</td>
<td>.094</td>
<td>.244</td>
<td>2.020*</td>
<td>.046</td>
</tr>
<tr>
<td></td>
<td>$\Delta F (2, 87) = 2.388$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tasks of multiple representations</td>
<td>$p$ equals; 098</td>
<td>.049</td>
<td>-.036</td>
<td>-.298</td>
<td>.766</td>
</tr>
</tbody>
</table>

*Note. *$p < .05$, two-tailed.

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4When inclusion of a predictor variable increases a regression coefficient of another predictor in a multiple regression analysis, the included predictor is called the **suppressor variable** because it statistically “suppresses” irrelevant variance that it shares with the other predictor, not with the dependent variable, thereby enhancing the association between the other predictor and the dependent variable (Pedhazur, 1997). In our case, Teachers’ Pursuing Multiple Solution Methods functions as the suppressor variable, and its inclusion in the regression model led to the improvement in the prediction.
### Stepwise Regression Analyses Predicting Teacher Questions from the Task Variables, and Teachers’ Pursuing Multiple Solution Methods

<table>
<thead>
<tr>
<th>Predictors</th>
<th>Teachers’ Lower Order Questions</th>
<th>Teachers’ Higher Order Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R² change (ΔR²) and F change (ΔF)</td>
<td>Std Error</td>
</tr>
<tr>
<td>1st step</td>
<td></td>
<td>.434</td>
</tr>
<tr>
<td>Tasks of high cognitive demand</td>
<td>ΔR² = .040</td>
<td>.182</td>
</tr>
<tr>
<td></td>
<td>∆F(2,87) = 1.822 p = .168</td>
<td>.225</td>
</tr>
<tr>
<td>Tasks of multiple representations</td>
<td></td>
<td>.035</td>
</tr>
<tr>
<td>2nd step</td>
<td></td>
<td>.430</td>
</tr>
<tr>
<td>Tasks of high cognitive demand</td>
<td>ΔR² = .070</td>
<td>.6720</td>
</tr>
<tr>
<td></td>
<td>∆F(1,86) = 6.720 p = .011</td>
<td>.218</td>
</tr>
<tr>
<td>Tasks of multiple representations</td>
<td></td>
<td>.044</td>
</tr>
<tr>
<td>Pursuing multiple solution methods</td>
<td></td>
<td>.479</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.393</td>
</tr>
</tbody>
</table>

Note. *p < .05, 2-tailed. **p < .01, 2-tailed. ***p < .001, 2-tailed.
and students that illustrates an association of high cognitive demand tasks and teacher’s higher order questions.

Classroom Episode 2

1. Teacher: We have found the least common multiples for many pairs of numbers.
2. For example, 72 is the least common multiple for 24 and 36. Now, let’s
3. see what is a relation between the least common multiple and common
4. multiples for the two numbers, such as 24 and 36?
5. Student 1: 72 is the least common multiple of 24 and 36. 144 is a common multiple
6. of 24 and 36. 144 is from 72 being multiplied by 2.
7. Teacher: Good. Then, would you explain the relation between the least common
8. multiple and the common multiple of the two numbers?
9. Student 1: The common multiple 144 twice as much as the least common multiple.
10. Teacher: Have you heard what he said?
11. All Students: Yes.
12. Teacher: Is what he said correct?
13. All Students: Yes.
14. Teacher: Yes, 144 twice as much as the LCM. Then, what special term you can use
15. to indicate the relation?
16. Student 1: The common multiple (144) is a multiple of the least common
17. multiple 72.
18. Teacher: Very good.
19. Teacher: Ok. When we know the least common multiple of two numbers, we
20. can generate many common multiples of the least common multiple.
21. Can everyone please generate common multiples for the two numbers?
22. Write down the common multiples you generate.
23. How do you generate the common multiples?
24. Student 2: The least common multiple for 24 and 36 is 72. By multiplying 72
25. by 2 once, twice, the third time, or more times, we get 144, 288, 576 and
26. so on, many common multiples.
27. Teacher: Are you able to generate and list out all common multiples for
28. the two numbers?
29. Student 2: I can generate more of them.
30. Teacher: Yes, we can continuously generate more common multiples of the two
31. numbers. Then, what do you think if there is the largest common multiple
32. which we can come up with?
33. Student 3: We can get many many more common multiples of the two numbers,
34. it seems we always can get another one, another larger one . . . .
35. Teacher: Whether or not do you think we can get to the largest common multiple
36. for two numbers?
37. Student 3: It’s like we can always get a larger one, another larger one . . . .
38. Teacher: You see, it is endless, we always can get a larger common multiple of two
39. numbers. Therefore there is no largest common multiple, but there is the
40. least common multiple of two numbers.

The task asked the students to draw the mathematical relation between the least common multiple and the common multiples of two numbers and then to infer whether there is a largest
common multiple of two numbers. Therefore, the task was considered one with high cognitive demand, procedure with connections. In the episode, the teacher asked higher order analytical questions (line 7–8, 14–15, 27–28, 31–32, 35–36) to guide the students to examine the mathematical relationship. The teacher raised specific, analytical questions (line 7–8, 14–15) about what is the mathematical relation between the least common multiple (72) and the common multiple when Student 1 indicated that 144 was generated by multiplying 72 by 2 (line 9). After asking the procedural simple questions (line 21, 23) to the students to generate common multiples for 24 and 36, the teacher pursued the connected higher order questions (line 27–28, 31–32, 35–36) that guided the students to understand that there is no largest common multiple.

The results of the regression analysis also indicated that Teachers’ Pursuing Multiple Solution Methods predicted Teachers’ Simple Questions. The following classroom episode illustrates this association.

Classroom Episode 3
(The teacher and the students were reading a question on the text book: “The size of the land in Asia accounts for one third of the total size of the land on the earth, it is one fifths and 3/25 for the size of the land in Africa and South America respectively. Which continent has the largest land size and which has the smallest?”)

1. Teacher: How would you figure out the answer? Who can find a fast way to do it? (The teacher asked one student to come to do it on the blackboard and the others do it at desk. The student was working on his answer on the blackboard: 1/3 = 25/75; 1/5 = 15/75; 3/25 = 9/75 25/75 > 15/75 > 9/75)
2. Teacher: Could you tell us your method to reach the answer? (referring to the student at the blackboard)
3. Student 1: The three fractions involve the mutually prime numbers. I have reduced
4. the fractions into their lower forms, multiplying one third with 25 . . . .
5. Teacher: Did you make the fractions into lower forms?
6. Students at desk: No, no, he changed the fractions into the higher forms.
7. Teacher: Yes, it is the common multiple. How did you find the common multiple?
8. Student 1: The size for South America is 3/25, it is one third for Asia. 3/25
9. one third are mutually prime numbers. Therefore, their least common
10. multiple (of the denominator) is 75.
11. Teacher: Is he right that the least common denominator 75?
12. Students: Yes.
13. Teacher: I see. You know 25 is a multiple of 5. Therefore, the common multiple
14. of 3 and 25 must be a multiple of 5 as well.
15. What did you do next?
16. Student 1: Comparing the three fractions with the common denominator.
17. 25/75 is the largest and 9/75 is the smallest.
18. Teacher: Have you got the same result? (Asking the class)
20. Teacher: Good. Any other method?
21. Student 2: I did this: 25 is 5 timing 5 and I changed one fifths into 5/25. 5/25 is
22. bigger than 3/25. Therefore, the size for Africa 5/25 is larger than that for
23. South America 3/25. Next, I compared one third for Asia with one fifths
24. for South America, one third is bigger than one fifths. Therefore, Asia has
This task was coded as a procedure without connections because it only involved comparing fractions. Comparing fractions by transforming them into common denominator form mainly applies a procedure known to most of the fifth graders. But the teacher also pursued different solution methods (line 20) from the students. The teacher asked several procedural simple questions (lines 2, 7, 15). The teachers then raised confirmation simple questions (lines 11, 18) to confirm with Student 1 and the class respectively about the procedures to find the common denominator and the answer to the question. After Student 2 provided a different method to reach the answer, the teacher invited confirmation from the class, “Is her method good?” (line 26). The teacher then evaluated the second method as simpler and praised the students for devising different methods (line 28–29). We often observed that the teachers were very encouraging with simple questioning when they pursued multiple solution methods and received positive responses from their students.

Predicting Student Answers: Simple and Highly Participatory Answers. Stepwise multiple regression models were also developed to predict Students’ Simple Answers. The predictors, Tasks of High Level Cognitive Demand and Tasks of Multiple Representations were entered at the first step, Teachers’ Pursuing Multiple-Solution Methods at the second step, and Teachers’ Lower Order Questions and Teachers’ Higher Order Questions at the third step. The results for this regression analysis are given in Table 5.

The two task variables at the first step resulted in a significant R-square change of 8.4% ($\Delta F(2, 87) = 3.971, p < .05$). Tasks of High Cognitive Demand positively predicted Students’ Simple Answers ($\beta = .308, p < .05$). The inclusion of Teachers’ Pursuing Multiple-Solution Methods into the regression model at the second step led to a further significant R-square change of 6.4% ($\Delta F(1, 86) = 6.487, p < .05$). In addition to Tasks of High Cognitive Demand, Teachers’ Pursuing of Multiple Solution Methods positively predicted Students’ Simple Answers ($\beta = .260, p < .05$). Adding Teachers’ Lower Order Questions and Teachers’ Higher Order Questions at the third step caused another significant R-square change of 19.1% ($\Delta F(2, 84) = 12.122, p < .001$). The full regression model indicates that only Teachers’ Lower Order Questions positively predicted Students’ Simple Answers ($\beta = .446, p < .001$).

The same stepwise multiple regression was carried out for Students’ Highly Participatory Answers. The variables in the first and second step were not significant. However, the entrance of Teachers’ Lower Order Questions and Teachers’ Higher Order Questions at the third step produced a significant R-square change of 18.5% ($\Delta F(2, 84) = 9.604, p < .001$). The regression model at this step shows that only Teachers’ Higher Order Questions positively predicted Students’ Highly Participatory Answers ($\beta = .451, p < .001$).

The results showed that different kinds of student responses were associated with different types of teacher questions, which in turn were associated with the task properties and Teachers’ Pursuit Of Multiple Solution Methods.
<table>
<thead>
<tr>
<th>Predictors</th>
<th>Students' Simple Answers</th>
<th></th>
<th></th>
<th>Students' Highly Participatory Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R² change (ΔR²) and F change (ΔF)</td>
<td>Std Error</td>
<td>Beta</td>
<td>t</td>
</tr>
<tr>
<td>1st step</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tasks of high</td>
<td>ΔR² = .084</td>
<td>.714</td>
<td>.308</td>
<td>2.592*</td>
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<tr>
<td>cognitive demand</td>
<td>ΔF (2, 87) = 3.971* p = .022</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Tasks of multiple</td>
<td>.370</td>
<td>-.042</td>
<td>-.352</td>
<td>.726</td>
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<tr>
<td>representations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd step</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tasks of high</td>
<td>ΔR² = .064</td>
<td>.708</td>
<td>.244</td>
<td>2.073*</td>
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<tr>
<td>cognitive demand</td>
<td>ΔF (1, 86) = 6.487* p = .013</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Tasks of multiple</td>
<td>.359</td>
<td>-.032</td>
<td>-.281</td>
<td>.779</td>
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<td>representations</td>
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<tr>
<td>Pursuing multiple</td>
<td>.788</td>
<td>.260</td>
<td>2.547*</td>
<td>.013</td>
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<td>solution methods</td>
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</tr>
<tr>
<td>3rd step</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tasks of high</td>
<td>ΔR² = .191</td>
<td>.648</td>
<td>.175</td>
<td>1.625</td>
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<tr>
<td>cognitive demand</td>
<td>ΔF (2,84) = 12.122*** p = .000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tasks of multiple</td>
<td>.321</td>
<td>-.056</td>
<td>-.543</td>
<td>.589</td>
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<tr>
<td>Pursuing multiple</td>
<td>.761</td>
<td>.158</td>
<td>1.598</td>
<td>.114</td>
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<td>solution methods</td>
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<tr>
<td>Teachers' lower</td>
<td>.160</td>
<td>.446</td>
<td>4.689***</td>
<td>.000</td>
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<td>order questions</td>
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<tr>
<td>Teachers' higher</td>
<td>.195</td>
<td>.072</td>
<td>.758</td>
<td>.451</td>
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<tr>
<td>order questions</td>
<td></td>
<td></td>
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</tbody>
</table>

*Note. *p < .05, 2-tailed. **p < .01, 2-tailed. ***p < .001, 2-tailed.*
Predicting Teachers’ Exploratory Responses. In the stepwise multiple regression model that analyzed Teacher’s Exploratory Responses to student answers, Tasks of High Cognitive Demand, and Tasks of Multiple Representations were entered as the predictors at the first step, Teachers’ Pursuing of Multiple Solution Methods at the second step, and Teachers’ Lower Order Questions, Teachers’ Higher Order Question, Student’s Simple Answers and Students’ Highly Participatory Answers at the third step. Results of the regression analysis are displayed in Table 6.

The regression model at the first step was significant with R-square change of 23.5% ($\Delta F(2, 87) = 13.342, p < .001$), where both Tasks of High Cognitive Demand ($\beta = .259, p < .05$) and Task of Multiple Representations ($\beta = .300, p < .01$) positively predicted Teachers’ Exploratory Responses. The regression model at the second step was not significant, indicating that Teachers’ Pursuing Multiple Solution Methods did not predict Teachers’ Exploratory Responses. Adding Teachers’ Lower Order Questions, Teachers’ Higher Order Questions, Students’ Simple Answers, and Students’ Highly Participatory Answers at the third step brought about a significant R-square change of 13% ($\Delta F(4, 82) = 4.430, p < .01$). The results from this step showed that Students’ Highly Participatory Answers positively predicted Teachers’ Exploratory Responses ($\beta = .282, p < .01$) in addition to the positive predictors, Tasks of High Cognitive Demand ($\beta = .250, p < .05$) and Tasks of Multiple Representations ($\beta = .301, p < .01$). However, Teachers’ Pursuing Multiple

<table>
<thead>
<tr>
<th>Predictors</th>
<th>$\Delta R^2$ change ($\Delta R^2$) and F change ($\Delta F$)</th>
<th>Std Error</th>
<th>Beta</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st step</td>
<td>Tasks of high cognitive demand</td>
<td>$\Delta R^2 = .235$</td>
<td>.353</td>
<td>.259</td>
<td>2.382*</td>
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<tr>
<td></td>
<td>$\Delta F(2, 87) = 13.342**$</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>$p = .000$</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td></td>
<td>Tasks of multiple representations</td>
<td>.183</td>
<td>.300</td>
<td>2.759**</td>
<td>.007</td>
</tr>
<tr>
<td>2nd step</td>
<td>Tasks of high cognitive demand</td>
<td>$\Delta R^2 = .033$</td>
<td>.356</td>
<td>.304</td>
<td>2.783**</td>
</tr>
<tr>
<td></td>
<td>$\Delta F(1, 86) = 3.878$</td>
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</tr>
<tr>
<td></td>
<td>$p = .052$</td>
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<tr>
<td></td>
<td>Tasks of multiple representations</td>
<td>.180</td>
<td>.293</td>
<td>2.740**</td>
<td>.007</td>
</tr>
<tr>
<td></td>
<td>Teachers’ pursuit of multiple solution methods</td>
<td>.396</td>
<td>−.187</td>
<td>−1.969</td>
<td>.052</td>
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<tr>
<td>3rd step</td>
<td>Tasks of high cognitive demand</td>
<td>$\Delta R^2 = .130$</td>
<td>.345</td>
<td>.250</td>
<td>2.354*</td>
</tr>
<tr>
<td></td>
<td>$\Delta F(4, 82) = 4.430**$</td>
<td></td>
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<tr>
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<td>$p = .003$</td>
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<td></td>
<td>Tasks of multiple representations</td>
<td>.169</td>
<td>.301</td>
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<td>.055</td>
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<td>Students’ highly participatory answers</td>
<td>.073</td>
<td>.282</td>
<td>2.908**</td>
<td>.005</td>
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</table>

*Note. *$p < .05$, 2-tailed. **$p < .01$, 2-tailed. ***$p < .001$, 2-tailed.
Solution Methods showed a tendency to predict negatively Teachers’ Exploratory Responses ($\beta = -0.187, p = .052$ at the second step; $\beta = -0.167, p = .088$ at the third step), though this relationship was not at the significant .05 level. Teachers’ Higher Order Questions was not indicated to be a significant predictor of Teachers’ Explorative Responses. The intercorrelation between two or more predictors indicates that the predictors share a certain amount of redundant information, which may lead one or more of the variables’ regression coefficients to be not statistically significant when the regression coefficient of a variable is tested separately (Pedhazur, 1997).

In Classroom Episode 4, the teacher explored a student’s response by asking her to illustrate her thinking process through drawing.

Classroom Episode 4
(The teacher and the class are working on a word problem involving fraction numbers, “A month has nine weekend days. The nine days take up 3/10 of the total days in that month. How many days are there in that month?”)

1. Teacher: What do you learn from the question? Chang (student name)?
2. Student 1: I learn that the month has 9 weekend days. It is 3/10 of the total days in that month. The question is asking how many days the month has.
3. Teacher: How do you understand “The nine days take up 3/10 of the total days in that month”?
4. Student 1: It means that these 9 days equal 3/10 of the total days.
5. Teacher: The 3/10 of the total days in that month is?
6. Student 1: 9 days.
7. Teacher: Good. Sit down. Do you know how to solve this problem?
8. Students: Yes!
9. Teacher: Good! Write your work in your excises booklets. (after a while, the teacher invites a student to explain her answer.)
10. Student 2: Nine days is the 3/10 of this month, I used 9 to be divided by 3 to get 3. 3 days is 1/10 of the month.
11. Teacher: Can you draw a graph to illustrate your solution? (The student draws a graph on the blackboard and the teacher asks the student to explain the draw.)
12. Student 2: I divided the month into 10 portions. Three portions are 9. So I can use 9 to be divided by 3 to get one portion, it is three days. Three days a portion. All together are 10 portions. The answer is 30 days.
13. Teacher: Do you understand? (asking the class)

The task was coded as a procedure with connections because it required the students to formulate a mathematical relation between the given quantities and to find out the unknown from the known quantities by applying the procedure. Word problems involving fraction numbers with division or multiplication were shown to be cognitively challenging for school children (Thompson & Saldanha, 2003). As shown in lines 1–8, the teacher used the several descriptive questions (Simple Questions) to ask the students to describe the literal meaning of the problem and to make sure the students would understand the question literally, which was typical in many of the Chinese classrooms working on word problems. Student 2 then provided a highly participatory answer.
by explaining how she solved this mathematic problem (line 12–13) upon the teacher’s request. This gave the teacher an opportunity to press the student to illustrate her thoughts with a drawing (line 14). The teacher then took the opportunity to invite the class for further discussion by attending to the student’s visual illustration and explanation of the solution (line 15–18). Episode 4 demonstrates how a task of higher cognitive demand encouraged a student’s highly participatory answer, which in turn motivated a teacher’s explorative response.

**Predicting Evaluation Authority: Teacher Alone and Teacher–Students Jointly.** Step-wise multiple regression models were used to identify the variables that predict Teacher Authority or Teacher–Student Joint Authority. Tasks of High Cognitive Demand and Tasks of Multiple Representations were entered at the first step, Teachers’ Pursuing Multiple Solution Methods was entered at the second step, and Teachers’ Lower Order Questions, Teachers’ Higher Order Question, Student’ Simple Answers, Students’ Highly Participatory Answers and Teachers’ Exploratory Responses were entered at the third step. Results of the regression analyses are provided in Table 7.

The regression model at the first step was significant, with R-square change of 25.2\% (ΔF(2, 87) = 14.664, p < .001). The variable of Tasks of High Cognitive Demand (β = .434, p < .001) was a strong and positive predictor for Teacher Authority in class discourse. However, the variable of Tasks of Multiple Representations was not a significant predictor. This might in part have to do with the significant correlation between the two predictors (r = .504, see Table 2). Teachers’ Pursuing Multiple Solution Methods at the second step did not result in a statistically significant R-square change. Instead, the entrance of the discourse variables at the third step produced a significant R-square change of 25.5\% (ΔF(5, 81) = 8.448, p < .001). Teachers’ Higher Order Questions (β = .400, p < .001) and Teachers’ Exploratory Responses (β = .334, p = .001) were positively predicting Teacher Authority, whereas Students’ Highly Participatory Answers (β = −.233, p < .05) negatively predicted Teacher Authority.

The regression models for predicting Teacher–Student Joint Authority at both the first and the second steps were not significant. However, the entrance of related discourse variables at the third step resulted in a statistically significant R-square change of 34.2\% (ΔF(5, 81) = 9.247, p < .001). Teachers’ Lower Order Questions (β = .277, p < .05), Students’ Highly Participatory Answers (β = .256, p < .05), and Teachers’ Exploratory Responses (β = .408, p < .001) significantly and positively predicted Teacher–Student Joint Authority.

Classroom Episode 2 above, where the teacher and students worked on the relation between the least common multiple and the common multiples of two numbers also illustrates the associations among the high cognitive demand task, the teacher’s higher order questions, and the teacher’s authority in evaluating the students’ answers.

Excerpt from Classroom Episode 2

5. Student 1: 72 is the least common multiple of 24 and 36. 144 is a common multiple of 24 and 36. 144 is from 72 being multiplied by 2.
6. Teacher: Good. Then, would you explain the relation between the least common multiple and the common multiple of the two numbers?
### Table 7

Stepwise Regression Analyses Predicting Authority in Evaluating Students' Responses From the Task Variables, Teachers' Pursuing Multiple Solution Methods, Teachers' Questions, Students' Answers, and Teachers' Exploratory Responses

<table>
<thead>
<tr>
<th>Predictors</th>
<th><strong>Teacher Authority</strong></th>
<th></th>
<th><strong>Teacher–Student Joint Authority</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R^2 ) change and ( \Delta R^2 ) change ( (\Delta F) )</td>
<td>Std Error</td>
<td>Beta</td>
<td>t</td>
</tr>
<tr>
<td>1st step</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tasks of high cognitive demand</td>
<td>( \Delta R^2 = .252 )</td>
<td>.226</td>
<td>.434</td>
<td>4.039***</td>
</tr>
<tr>
<td></td>
<td>( \Delta F(2,87) = 14.664*** )</td>
<td>( p = .000 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tasks of multiple representations</td>
<td>.117</td>
<td>.116</td>
<td>1.081</td>
</tr>
<tr>
<td>2nd step</td>
<td>Tasks of high cognitive demand</td>
<td>( \Delta R^2 = .004 )</td>
<td>.232</td>
<td>.449</td>
</tr>
<tr>
<td></td>
<td>( \Delta F(1,86) = .425 )</td>
<td>( p = .516 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tasks of multiple representations</td>
<td>.118</td>
<td>.114</td>
<td>1.056</td>
</tr>
<tr>
<td></td>
<td>Pursing multiple solution methods</td>
<td>.258</td>
<td>.062</td>
<td>-.652</td>
</tr>
<tr>
<td>3rd step</td>
<td>Tasks of high cognitive demand</td>
<td>( \Delta R^2 = .255 )</td>
<td>.209</td>
<td>.218</td>
</tr>
<tr>
<td></td>
<td>( \Delta F(5,81) = 8.448*** )</td>
<td>( p = .000 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tasks of multiple representations</td>
<td>.104</td>
<td>-.010</td>
<td>-.107</td>
</tr>
<tr>
<td></td>
<td>Pursing multiple solution methods</td>
<td>.242</td>
<td>.055</td>
<td>.617</td>
</tr>
<tr>
<td></td>
<td>Teachers' lower order questions</td>
<td>.056</td>
<td>-.030</td>
<td>-.316</td>
</tr>
<tr>
<td></td>
<td>Teachers' higher order questions</td>
<td>.067</td>
<td>.400</td>
<td>4.259***</td>
</tr>
<tr>
<td></td>
<td>Students' simple answers</td>
<td>.034</td>
<td>.156</td>
<td>1.605</td>
</tr>
<tr>
<td></td>
<td>Students' highly participatory answers</td>
<td>.045</td>
<td>-.233</td>
<td>-2.531*</td>
</tr>
<tr>
<td></td>
<td>Teachers' exploratory responses</td>
<td>.065</td>
<td>.334</td>
<td>3.340**</td>
</tr>
</tbody>
</table>

Note. *\( p < .05 \), 2-tailed. **\( p < .01 \), 2-tailed. ***\( p < .001 \), 2-tailed.
9. Student 1: The common multiple 144 twice as much as the least common multiple.
10. Teacher: Have you heard what he said?
11. All Students: Yes.
12. Teacher: Is what he said correct?
13. All Students: Yes.
14. Teacher: Yes, 144 twice as much as the LCM. Then, what special term you can use to indicate the relation?
15. Student 1: The common multiple (144) is a multiple of the least common multiple 72.
17. Teacher: Are you able to generate and list out all common multiples for the two numbers?
18. Student 2: I can generate more of them.
19. Teacher: Yes, we can continuously generate more common multiples of the two numbers.

In the episode, this teacher was shown to be able to engage the students in the mathematical discussion of a high cognitive demand task with higher order questioning. The teacher’s authority in evaluating the students’ responses was evident as the teachers gave the definite confirmation of the student responses four times in the exchanges (line 7, 14, 18, 30–31).

In the current reform in Mainland China, teacher–student joint authority in evaluation is greatly encouraged. Although some teachers did manage to initiate student-oriented conversation in their classes to enhance high level reasoning, many mathematical teachers involved teacher–student authority in evaluating simple answers, as illustrated in Classroom Episode 5 below. It exemplifies the observed association of teachers’ lower order questions with teacher–student joint authority in evaluating students’ answers.

Classroom episode 5
The teacher wrote on the blackboard:

1) How much is the 2/3 of 5?
2) How much is the 1/3 of 10?
3) How many is the 3/5 of the whole class?

1. Teacher: Let’s look at the problems. Please write down the number sentences but not calculate them.
2. (Students are writing. Those who have finished sit in an upright way.)
3. Teacher: How much is the two third of 5, Yang?
4. Student 1: 5 multiplies two third.
5. Teacher: Is he right?
7. Teacher: How much is the one third of 10, Li?
8. Student 2: 10 multiplies one third.
9. Teacher: Do you agree?
10. Students: Yes.
11. Teacher: The third question, How much is the three fifths of the whole class?
12. Student 3: The total number of the class multiplies three fifths.
13. Teacher: Do you agree?

In Classroom Episode 5, the teacher posed simple questions on how to transform the corresponding questions into mathematical number sentences three times (lines 3, 7, and 11). Then based on students’ answers, the teacher invited the whole class to evaluate those simple replies by asking, “Is he right?” or “Do you agree?” (lines 5, 9, 13). This appeared to be a simple and common way that many of the Chinese teachers used to encourage their students to evaluate the answers of peers in our classroom data.

**Comparative Results Concerning 30 Classrooms as the Unit of Analysis.** The same stepwise regression models applied to the 90 classrooms were also tested with the dataset of the 30 classrooms. The following results involving 30 classrooms were consistent with those obtained with the 90 class sessions. First, Teachers’ Higher Order Questioning positively predicted Students’ Highly Participatory Answers, whereas Teachers’ Lower Order Questions positively predicted Students’ Simple Answers. Secondly, Teachers’ Simple Questions as well as Students’ Highly Participatory Answers positively predicted Teacher–Student Joint Authority over evaluating student responses. Thirdly, Tasks of High Cognitive Demand as well as Teachers’ Higher Order Questions positively predicted Teacher Authority over classroom discourse.

However, there were also some inconsistencies between the results from the 30 classrooms and those from 90 class sessions. First, the analysis with the 90 class sessions (see Table 3) indicated that the regression coefficient of Tasks of High Cognitive Demand was significant for predicting Teachers’ Pursuing Multiple Solution Methods, but it was not observed in the analysis with the 30 classrooms. Secondly, in the analysis with the 30 classrooms, Tasks of Multiple Representations was shown to be a positive predictor of Teachers’ Higher Order Questioning, whereas with the dataset of 90 class sessions Tasks of High Cognitive Demand was the positive predictor. This probably had to do with the moderate level of correlation ($r = 0.504$) between the two task variables. Thirdly, the analysis of 90 class sessions indicated that Teachers’ Pursuing Multiple Solution Methods positively predicted Teachers’ Lower Order Questions, but the analysis with 30 classrooms did not show this. Lastly, the analysis with the 30 classrooms did not indicate any significant predictor for Teachers’ Exploratory Responses. In contrast, the analysis with 90 class sessions indicated that Tasks of High Level Cognitive Demands, Tasks of Multiple Representations, as well as Students’ Highly Participatory Answers positively predicted Teachers’ Exploratory Responses. The inconsistencies mainly were about those positive associations that were shown in the analyses using the 90 classes but absent involving the 30 classrooms. It was understood that the aggregated classroom data (30 classrooms) might diminish the variances of the observed variables in the individual classes, which might thereby weaken the associations among the variables.

**DISCUSSION**

The present study investigated the relations between the features of mathematical tasks, teachers’ pursuit of multiple solution methods, and the features of classroom discourse in the Chinese
classrooms that were implementing the reformed mathematics curriculum. The results of the study contribute to our understanding of the rich and complex relations between task variables and discourse variables from a new and different cultural setting.

Main Findings and Their Implications

The study tested the five sets of hypotheses concerning the relations of the task variables to the discourse variables. The subsections that follow in order will discuss the findings to address each set of the hypotheses.

**Task Properties in Relation to Teachers’ Pursuing Multiple Solution Methods.** The regression model was not significant to predict teachers’ pursuit of multiple solution methods with the two task variables, high cognitive demand and multiple representations, although the individual regression coefficient for Tasks of High Cognitive Demand reached a significant level ($p = .046$). This result was not consistent with the theoretical speculation that tasks with multiple representations are likely to increase the possibility for the use of different methods to solve the problems (Brenner et al., 1997). One reason may involve the teacher’s purpose in pursuing multiple solution methods. As shown in some previous studies (Baxter & Williams, 2010; Williams & Baxter, 1996), teachers pursued multiple solution methods in mathematics classrooms more for social scaffolding to encourage students to talk than for cognitive scaffolding to advance a mathematics understanding. The present study shared the similar observation. The pursuit of multiple solution methods for social scaffolding might make teachers less evaluative of students’ responses and thus might have weakened its association with the two task properties. Another reason might be that most of the mathematics problems can be solved with multiple solution methods. This implies no necessary connection between pursuing multiple solution methods and tasks of high cognitive demand or multiple representations. Therefore, the question of whether or not there would be an association between either of the two task variables and the likelihood of teachers’ pursuing multiple solution methods becomes largely an empirical question, depending on the concerned teacher and particular context of his or her classroom.

In contrast, the two task variables were correlated substantially ($r = .504$). The correlation suggested that classroom tasks of high cognitive demand were more likely to lead to making connections among multiple representations.

**Task Properties, Teachers’ Pursuit of Multiple Solution Methods, in Relation to Teachers’ Higher Order and Lower Order Questions.** As expected, high cognitive demand tasks were shown to link to higher order questions from teachers. Tasks of high cognitive demand were also related to teachers’ simple questions, which was understandable in the context of the present study. It was typical in many of the Chinese mathematics classrooms that a teacher would first work with his or her students in making sure that a given word problem was literally understood and the relations implied were clarified before solving a task of word problem. The more complicated a task presented in a word problem appeared to students, as perceived by a teacher, the
more likely it was that teachers asked questions to clarify the literal meaning of the problem early in approaching to the task.

Contrary to what was hypothesized, teachers’ pursuit of multiple solution methods was associated with teachers’ using lower order questions. A closer inspection of the videotaped lessons showed that many of the mathematics teachers required students to report their solution methods by simply asking, “Any other methods (opinions)?” After hearing another approach, teachers typically asked questions to confirm the procedure of a method or remarked, “Good, any other methods (opinions)?” and continued. Teacher only occasionally gave their students sufficient time and attention to compare different approaches or to evaluate why one method was more efficient than the other. Baxter and Williams (2010) reported similar observation. They found that the American middle school teachers using a reform mathematics curriculum effectively provided social scaffolding to encourage students to participate but did not effectively provide analytical scaffolding to support students in processing information in a certain way to construct a particular concept or acquire a specific skill. This was observed as well in a language arts classroom where the teacher encouraged students to offer their views but seldom provided the opportunity for the students to evaluate their viewpoints (Turner, 1995).

In situations where a teacher’s priority was to get students to converse more, he or she might be less reflective of student responses. On the other hand, this might also indicate the teacher’s effort to create an atmosphere that is less evaluative in order to encourage students to talk more.

In the present study, the observed association of teachers’ pursuing multiple solution methods with teachers’ simple questions might also be related to an effect that was topic specific. For example, in the 11 class sessions on teaching reciprocals of the total 90 class sessions, none of those teachers pursued multiple solution methods for the tasks to find the reciprocals of given numbers. Conversely, among the nine class sessions on the least common multiples and comparing fractions, all of the teachers requested that the students use multiple methods to complete the tasks. In the Teacher’s Manual (Working Group for School Mathematics of Nine-Year Compulsory Education, 2001) for the new curriculum explicitly advises teachers to use multiple methods to compare sizes of fractions. The observation therefore has to be understood against the context and to be examined and validated in future study.

Task Properties, Teachers’ Pursuit of Multiple Solution Methods, Teachers’ Questions, in Relation to Students’ Answers. The results supported the hypothesized association of teachers’ higher order questions with students’ highly participatory answers, and that of teachers’ lower order questions with students’ simple answers. However, neither tasks of high cognitive demand or multiple representations or teachers’ pursuit of multiple solution methods had a direct association with students’ highly participatory answers. Moreover, both tasks of high cognitive demand and teachers’ pursuit of multiple solution methods were positively related to

5The observed content-specific effect associated with Teachers’ Pursuing Multiple Solution Methods appears to be contradictory with the statement that most of the mathematics tasks can be solved with more than one method but the statement does not logically exclude any of the observations.
students’ simple answers. However, only teachers’ lower order questions positively predicted students’ simple answers when the regression model included the predictors of teachers’ higher order questions and teachers’ lower order questions, in addition to the task-related variables (high cognitive demand and multiple representations). The results were logically consistent with the observed associations of the task-related variables with teachers’ questions as discussed in the preceding section. These findings were also consistent with Stein’s analysis (Stein et al., 1996, 2000) that the influence of instructional tasks on student learning is realized through a chain of task implementation. Various factors can affect the way in which an instructional task is enacted in actual classrooms, and the nature of the teacher questions is one of those factors.

Task Properties, Teachers’ Pursuit of Multiple Solution Methods, Teachers’ Questions, Students’ Answers, in Relation to Teachers’ Exploratory Responses. As expected, teachers were more likely to explore students’ answers when tasks of high cognitive demand or multiple representations were used and when the teachers required students to explain and analyze their answers. However, it was not expected that teachers would be less likely to make additional inquiries into students’ answers while they pursued multiple solution methods. As explained above, this might be related to the teacher’s purpose in pursuing multiple solution methods, if the teacher’s intention was to encourage students to talk more, and the teachers therefore were less evaluative of students’ responses.

Task Properties, Teachers’ Pursuit of Multiple Solution Methods, Teachers’ Questions, Students’ Answers, Teachers’ Exploratory Responses, in Relation to the Nature of Authority Over Classroom Discourse. The results showed a complicated picture of the relations between the variables that partially rejected our initial hypothesis. Consistent with the hypothesis, students’ highly participatory answers and teachers’ explorative responses positively predicted joint teacher–student authority over evaluation of student responses. However, contrary to the hypothesis, the teachers tended to be more dominant when they used tasks of high cognitive demand or raised higher order questions. Conversely, neither tasks of high cognitive demand, tasks of multiple representations, or teachers’ pursuit of multiple solution methods was associated directly with teacher–student joint authority over classroom discourse.

However, the results suggest plausible indirect links between teacher–student joint authority and tasks of high cognitive demand, as well as to teachers’ pursuing multiple solution methods. That is, there were connections from tasks of high cognitive demand to teachers’ higher order questions and from teachers’ higher order questions to students’ highly participatory responses, which in turn contributed to the creation of teacher–student joint authority. Teachers’ pursuit of multiple solution methods was associated with lower order questions from teachers, and teachers’ lower order questions were associated with instances of teacher–student joint authority in evaluating student responses.

The observed association of teacher authority with the tasks of high cognitive demand and teachers’ higher order questions as well as the association of teacher–student joint authority with teachers’ simple questions might reflect the difficulty teachers experience in managing
the multiple but not necessarily converging demands in teaching students mathematics in the Chinese classrooms. Engle and Faux (2006) showed that American teachers also had difficulty giving students voices while holding them accountable for how their contributions did or did not improve the disciplinary ideas under discussion. The teachers encouraged students to talk more but seldom offered students the opportunity to evaluate their thinking with respect to disciplinary ideas.

These results indicate that the specific characteristics of instructional tasks had complicated relations with the nature of classroom authority, teacher authority or teacher–student joint authority over knowledge (Hamm & Perry, 2002). Instructional tasks with high cognitive demand or multiple representations certainly provide richer and more demanding content that encourages discussions between teachers and students. Teachers’ pursuit of multiple solution methods was also supposed to facilitate this. However, the present results suggest that teachers’ emphasis on social scaffolding or on cognitive scaffolding might strongly affect the influence of instructional tasks on the nature of classroom discourse (Baxter & Williams, 2010; Turner, 1995; Williams & Baxter, 1996). This indicates the dominant role that a teacher can play in steering classroom discourse towards greater teacher authority or greater teacher–student shared authority. The teacher’s understanding of the purpose of an instructional task and his or her beliefs about the nature of mathematics teaching and about his or her students’ capability certainly affects the teacher’s role in mediating the relations of instructional tasks to classroom discourse. Therefore, it seems highly unlikely that a complete transformation of classroom practice to teacher–student shared authority would follow changes in instructional tasks alone.

**Issues and Limitations**

Some concerns with the present study and results should be noted. As explained in the Method section, the variable of Teachers’ Pursuing Multiple Solutions was coded in terms of number of tasks in which a teacher pursued multiple solution methods. This coding indicates only whether or not a teacher pursued multiple solution methods but not how frequently the teacher did so in a given task. The present results only showed the relationship of the variable to the other variables in the way it was coded. It remains unclear whether the observed relationships concerning this variable would be similar if it was be coded according to the frequency by which a teacher pursued multiple solution methods per task.

Discrete codes were used to analyze the features of instructional tasks and classroom discourse, and these could not adequately capture all richness and dynamics of classroom practice. For example, coding a task as having or not having multiple representations does not tell anything about whether the different representations are complementary to each other. In addition, classroom instruction involves a web of events, individuals, materials, and locations. Discrete codes segment such a web into components, and then a re-configuration of relations among them might be inferred and tested, as the present study did. It is difficult to know to what degree that re-configuration reflects the original complexity. Therefore, it is important to utilize and examine such data in different ways for verification and validation. For example, to verify how high cognitive demand tasks affect teacher–student discourse, a future study could analyze qualitatively and quantitatively the nature of classroom discourse following high cognitive demand tasks in comparison to following low demand tasks. As this study and other studies
have indicated, the teacher’s emphasis on social scaffolding or cognitive scaffolding influences what type of questions a teacher asks and thus moderates the influence of instructional tasks on the nature of classroom discourse. Therefore, it will be also of theoretical and empirical interest to understand the decision making processes of the teachers who are able to incorporate instructional tasks and teacher–student discourse in comparison to those who appear less able to do so.

For the purpose of the present study, teachers’ questions were classified into lower order questions and higher order questions, and students’ answers were classified into simple answers and highly participatory answers. It should be noted that the labels used are not value bound, and it is important to avoid the presumption that higher order questions or participatory answers are good and that lower order questions or simple answers are not. At both a theoretical and a practical level, these variables are not directional. Consequently, the utility of lower order questions or higher order questions and simple answers or highly participatory answers cannot be judged out of context.

In addition, the present study utilized the 90 individual class sessions, three sessions from each of the 30 classrooms, for the analyses because of the large variability in the measured variables between sessions within class. The correlational data on the three observations of the variables somehow alleviated the concern over the violation of the independence assumption. However, the caveat remains for the three variables (students’ highly participatory responses, teacher authority, teacher–student joint authority) that show significant correlations between Time 1 and Time 3 observations. Therefore, interpretation of the results should take this into consideration. That said, many of the results obtained from the dataset of 30 classrooms were consistent with those obtained with the dataset of 90 class sessions. The inconsistent results can mostly be attributed to findings that some of the significant associations between the variables observed with the 90 class sessions were not present in the analyses with the 30 classrooms because the aggregated classroom data (30 classrooms) might diminish the variances of the observed variables in the individual classes. Again, we considered that the class session data rather than the aggregated classroom data was more appropriate to examine the relations between the features of instructional tasks and classroom discourse to take into account the variability among the class sessions. Also, taking the sample size into consideration, we suggest that the results from the 90 class sessions were more plausible. These associations were derived from naturalistic situations where the 90 class sessions differed in teachers’ allocation of time to academic activities, in organization of the academic activities, and in student prior knowledge with respect to the taught topics. Therefore, they probably pointed to some patterns of association that are meaningful to inform the current instructional practice in Chinese classrooms.

Finally, it should be noted that the results of the study illustrated the situation of classroom instruction in the current enactment stages of the reformed curriculum. Most of the teachers in the present study had only one to three years of teaching experience with the new curriculum. For example, in a survey study (X. Q. Li, Ni, Li, & Tsoi, 2012), teachers with more years of experience implementing the reformed curriculum reported higher frequency of using tasks of high cognitive demand in their mathematics classrooms. Another practical constraint for those classrooms was the large class size. Most classrooms in the city where the study was conducted and those classrooms participating in the current study had an average class size of close to 60 students. The large class size probably posed a challenge for teachers to create more dialogic classrooms. Blatchford, Bassett, and Brown (2011) observed 88 classrooms from primary and
secondary schools whose class sizes varied around 15 to 30 students per class. They showed that larger class size was associated with fewer teacher and individual student interactions and more instances of the one-way direct instruction from the teacher to the whole class.

CONCLUSIONS

The present study describes the rich and complex relationships of instructional tasks to teacher–student discourse in the Chinese mathematics classrooms. The results suggest that the types of mathematics tasks and teachers choices affect the nature of engagement of a teacher with his or her students. Teachers may emphasize social scaffolding to encourage more students to talk and participate, cognitive scaffolding for students to achieve understanding of the important mathematics ideas, or may manage or balance the two. The implications of the findings do not appear new but are still significant with respect to the current literature, because they are illustrated in a new cultural setting. In addition, the findings add to the literature on the pedagogy of mathematics instructional tasks (e.g., Stein et al., 1996, 2000, 2008) and on complex relations of pedagogy to curriculum (Doyle, 1983, 1992; Stein et al., 2007) by providing a richer description about the relations of the specific features of mathematics instructional tasks to specific features of classroom discourse. They inform the current instructional practice in Chinese mathematics classrooms implementing the new curriculum and improve our understanding of those relations in multiple cultural contexts.

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