



香港中文大學 教育學院 大學與學校夥伴協作中心

CUSP (Mathematics) Workshop



HUMANISING MATHEMATICS:

MAKE MATHS MEANINGFUL THROUGH **SCT, SDL, PLM**

「人本數學：尋覓數學學習的意義」

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MATHEMATICS STORIES

故事（一）

我還清楚記得小學四年級數學課的挫敗經歷。那活該的4分鐘競賽活動，要求我們在4分鐘內完成200題乘數。成績素來優異的我，突然感到失敗、羞辱，縱使再嘗試，也是失敗。我感覺自己蠢，但我知道我不是愚笨，我祇是討厭那種感覺。最後，經過四次嘗試後，我成功了。但我已再不想看到我的名字在那恐怖的黃色告示板上出現，我完全感覺不到那次勝利的喜悅。我仍然很討厭那個計時器。

(WORD for Story 1: _____)

故事（三）

生命中我還記得第一次因為那個老師令我真正享受上數學課，而甚至獲得甲等成績。對我來說那是一個重要的轉捩點。

(WORD for Story 3: _____)

故事（二）

我記得在五年級要不斷地做解難的題目。其中最深刻的記憶，就是老師問了一個這樣的問題：「有3個人想過河，他們每人的重量皆為60 kg。但他們祇有一隻能負載120 kg的船，試問他們可以怎樣能讓全部人過到河？」

(WORD for Story 2: _____)

故事（四）

在學習幾何課上，我完全跟不上。我不能掌握其中的概念，亦「看」不到那些抽象的圖形和圖像。我的老師幫不了忙，祇懂得用一種方法教我們——單向式的講課，而從不改變他的教法。班上很多同學跟我一樣，感到迷惘。

(WORD for Story 4: _____)

數學意義的「尋」、「覓」



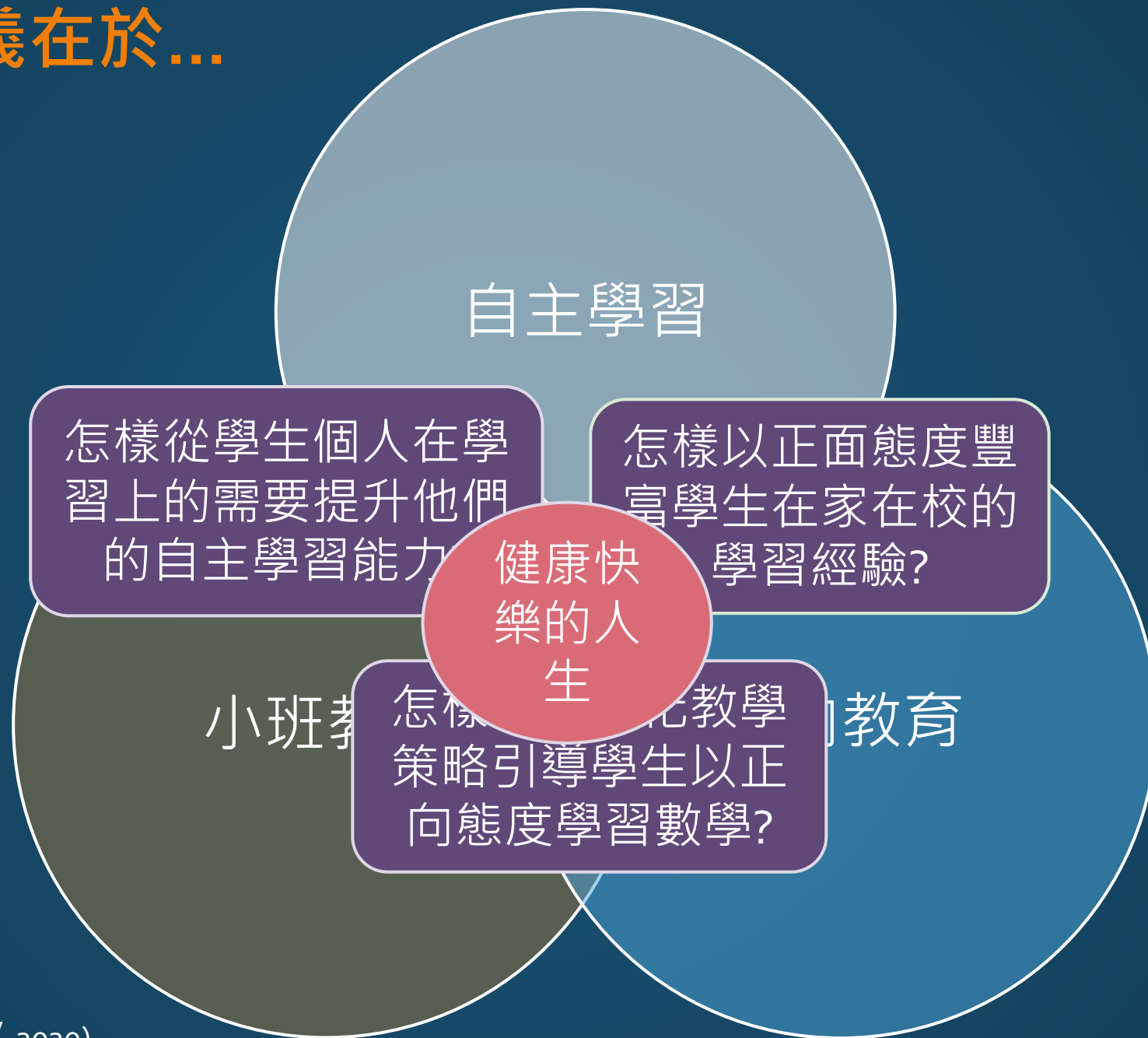
「尋」的本意，是量度人兩臂伸展開的長度

「尋」和「常」皆為古代的長度——八尺為尋，倍尋（一丈六尺）為常，喻短或小。「尋常」一詞，意指「普通、常見」

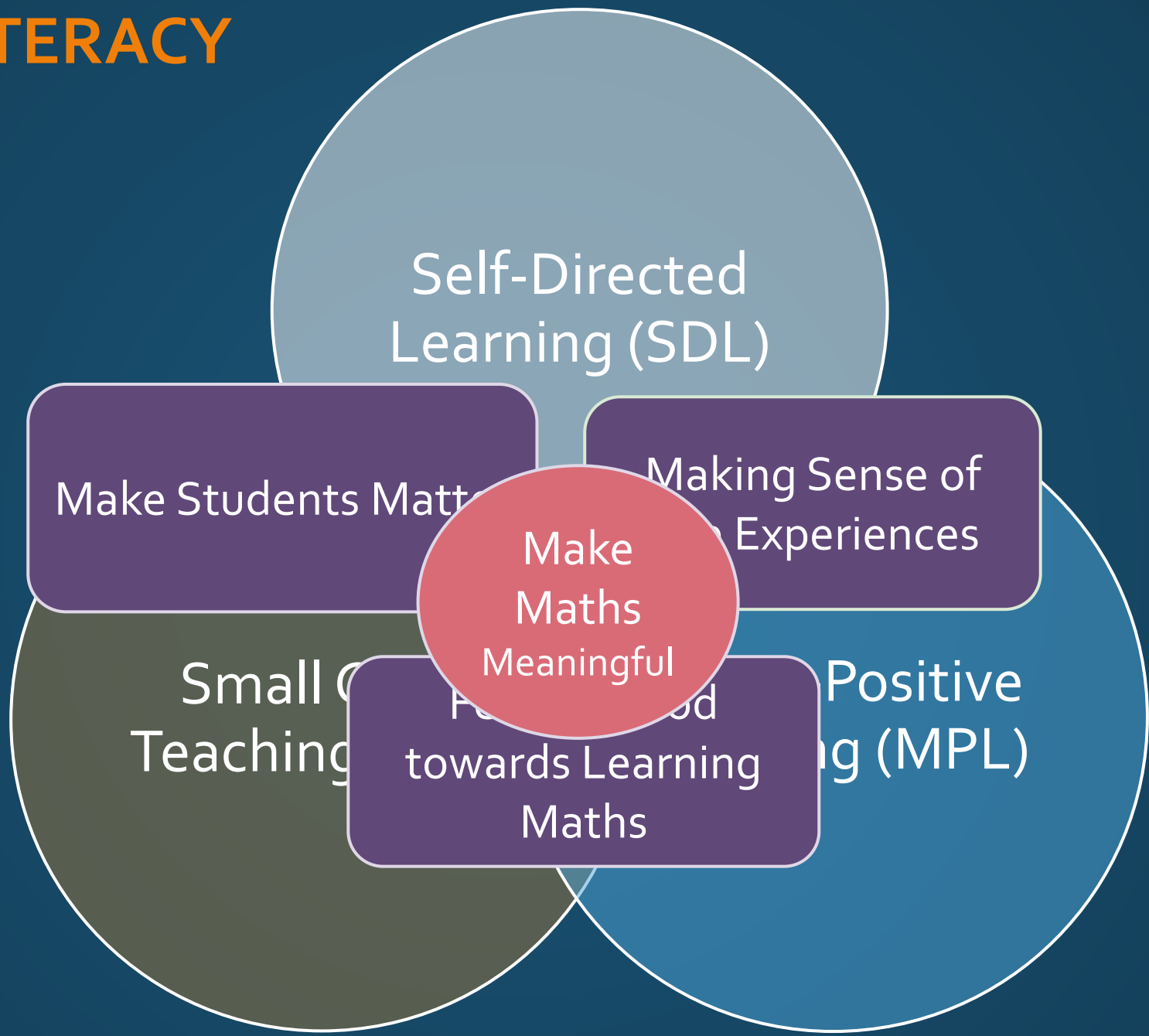


「覓」作為古文的形義，是從「見」從「爪」，以手半遮眼睛而視之形（見圖二），本義是窺視，引申表示尋求

學習數學的意義在於...



MATHS FOR LITERACY



Self-Directed Learning (SDL)

Make Students Matter

Making Sense of Experiences

Make Maths Meaningful

Small Steps towards Learning Maths

Positive Learning (MPL)

Positive Learning (MPL)

LEARNING MATHS FOR WELL-BEING

"YOU FEEL GOOD?"

(See Julia L. Hill, Margaret L. Kern, Wee Tiong Seah, and Jan van Driel's *Feeling Good and Functioning Well in Mathematics Education: Exploring Students' Conceptions of Mathematical Well-Being and Values*, 2021)

MATHEMATICS ATTITUDE: POSITIVE VS NEGATIVE

I like/dislike
Mathematics

Mathematics
is....

I can/can't do
it

(ZAN & DI MARTINO, 2003)

THINK OUT OF THE BOX



7
11

8
80

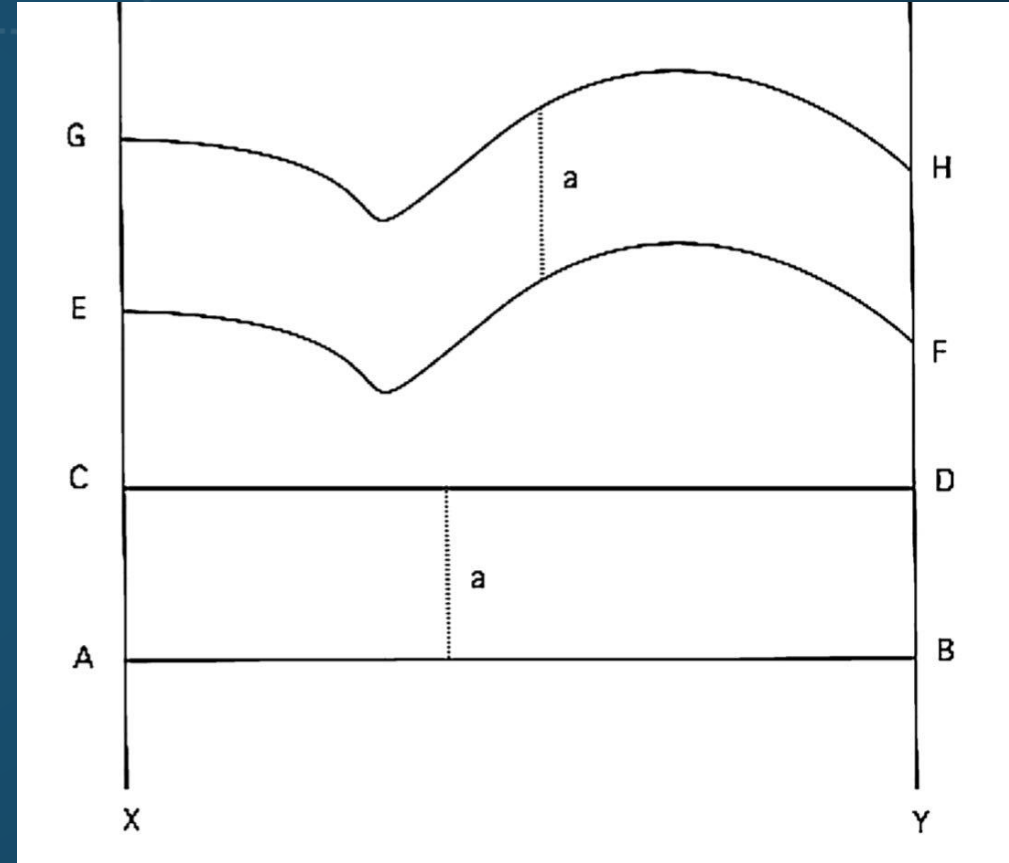
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LET'S CREATE **THOUGHTFUL DIALOGUE** IN THE MATHS LESSONS

Two students, Paul and Mary, were asked to compare the areas of the figures ABCD and EFGH (as shown).

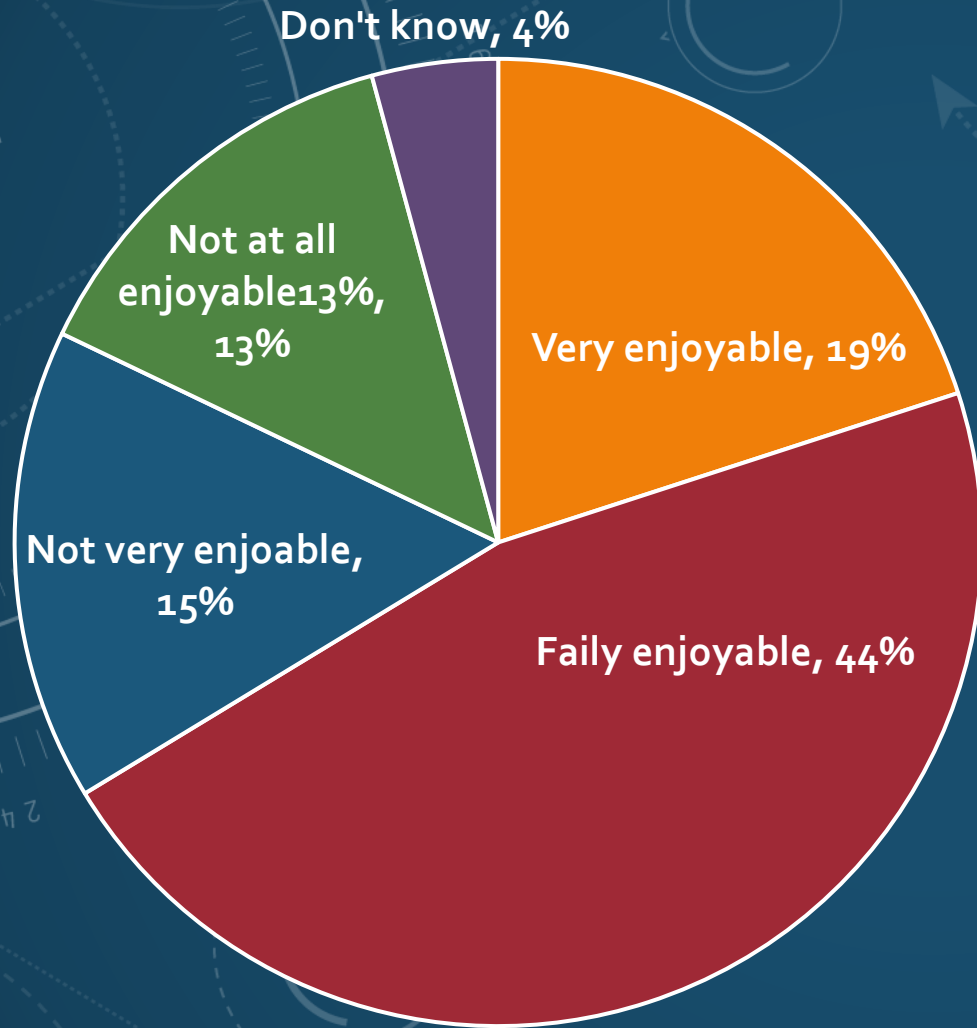
Mary affirmed that the area EFGH is greater than the area ABCD because EFGH was longer. Paul thought that the two areas should be equivalent and yet he could not offer Mary any convincing arguments for his statement.

What would you suggest them the ideas that you have?



Maths Enjoyment

HOW ENJOYABLE
WAS MATHS IN
PRIMARY SCHOOL?



BASE: 1129 ALL IN YEARS 7 OR 8, 2008
(IPSOS MORI, ENGLAND & WALES)

Very enjoyable Fairly enjoyable Not very enjoyable Not at all enjoyable 13% Don't know

MATHS LEARNER AS APPRENTICE TAILOR

Being a maths **learner** (tailor) is more than having a set of '**mathematising**' (tailoring) skills. It includes **a way of thinking, a way of seeing**, and having a set of **values and perspectives**.

(ALAN SCHOENFELD'S *IDEAS IN THE AIR: SPECULATIONS ON SMALL GROUP LEARNING, ENVIRONMENTAL AND CULTURAL INFLUENCES ON COGNITION, AND EPISTEMOLOGY*.1989, P. 85)

MAKING STUDENTS MATTER: SPECIAL LEARNER NEEDS (SLN)

REFER TO SPECIAL NEEDS OF STUDENTS WITH PARTICULAR **DETACHMENT FROM ACQUIRING MEANING IN LEARNING** A PARTICULAR SUBJECT SUCH AS **MATHEMATICS**

(HYLAW'S LOG, 1/8/2020)

How many students with SLN in the class you teach?

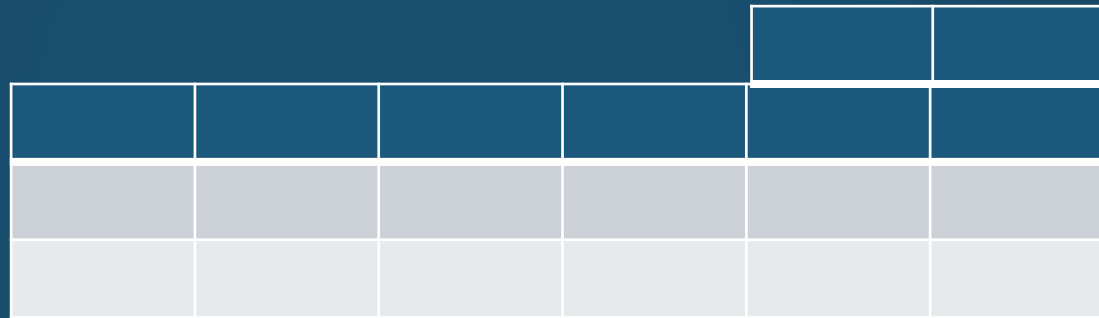
PERMA MODEL

1. **Positive emotion** (Positive emotions when learning/doing mathematics, such as enjoyment, fun, and happiness)
2. **Engagement** (A sense of concentration, absorption, deep interest, or focus when learning/doing mathematics)
3. **Relationships** (Having supportive relationships with others, believing one is valued and cared for, connected with others, or supporting peers in mathematics)
4. **Meaning** (Having a sense of direction in mathematics, feeling mathematics is valuable, worthwhile, or has a purpose)
5. **Accomplishment** (A sense of achievement, reaching goals, or mastery completing mathematical tasks and tests)

(See Norrish & Seligman, 2015)

HIDDEN MATHS CURRICULUM: POSITIVE MATHS LEARNING (PML)

Arrange 20 students in 6 groups



$$3x + 4y = 20, x + y = 6$$

$$3 < 20/6 < 4)$$

How about if we have 18 students

(See ABRAMOVICH & BROUWER, 2006)

CATEGORY	BUILDERS	BABY BOOMERS	GENERATION X	GENERATION Y	GENERATION Z	GEN ALPHA
Slang terms	 <p>We prefer proper English if you please</p> <p>Born: < 1946 Age: 74+</p>	 <p>Be cool Peace Groovy Way out</p> <p>Born: 1946-1964 Age: 55-73</p>	 <p>Dude Ace Rad As if Wicked</p> <p>Born: 1965-1979 Age: 40-54</p>	 <p>Bling Funky Doh Foshizz Whassup?</p> <p>Born: 1980-1994 Age: 25-39</p>	 <p>🔥 Fam GOAT Slay Yass queen</p> <p>Born: 1995-2009 Age: 10-24</p>	 <p>lit yeet hundo oof rn idrc</p> <p>Born: 2010-2024 Age: under 10</p>
Social markers	World War II 1939-1945	Moon landing 1969	Stock market crash 1987	September 11 2001	GFC 2008	Trump / Brexit 2016
Iconic cars	 Model T Ford Final, 1927	 Ford Mustang 1964	 Holden Commodore 1978	 Toyota Prius 1997	 Tesla Model S 2012	 Autonomous vehicles 2020s
Iconic toys	 Roller skates	 Frisbee	 Rubix cube	 BMX bike	 Folding scooter	 Fidget spinner
Music devices	 Record player LP, 1948	 Audio cassette 1962	 Walkman 1979	 iPod 2001	 Spotify 2008	 Smart speakers Now
Leadership style L - Leader l - New leaders	 Controlling	 Directing	 Coordinating	 Guiding	 Empowering	 Inspiring
Ideal leader	Commander	Thinker	Doer	Supporter	Collaborator	Co-creator
Learning style	Formal	Structured	Participative	Interactive	Multi-modal	Virtual
Influence/advice	Officials	Experts	Practitioners	Peers	Forums	Chatbots
Marketing	Print (traditional)	Broadcast (mass)	Direct (targeted)	Online (linked)	Digital (social)	In situ (real-time)

PRINCIPLES OF SMALL CLASS TEACHING

1. Clear **target** (clear to both teacher and students?)
2. Effective **questioning** (how teacher can guide students to do their own questioning?)
3. **Student engagement** (how to motivate more students to engage in the classroom activity?)
4. **Group collaborative learning** (how to promote interactive group dynamic?)
5. **Feedbacks** (would teacher's feedbacks inspiring enough to elicit students' responses?)
6. **Assessment for learning** (can it be made further to make assessment as learning?)
 - **Caring** for each learner as a **unique self** is the core principle!

MATHS-POSITIVE LEARNING: GROWTH MID-SET

1. Understand current mindsets
2. Talk positively about maths
3. Show that everyone can do maths
4. Celebrate mistakes
5. Praise carefully
6. Talk about learning

(Alexandra Riley's *Six Steps for a Positive Mindset in the Math Classroom*, 2020)

SELF-DIRECTED LEARNING: GROWTH

Experiencing mathematics

1. Seeing -- Questioning
2. Knowing -- Inquiring
3. Responding (Reflecting) -- Conceptualizing
4. Interacting (Participating) -- Experimenting

(See Linkins et al., 2015)

UNIVERSAL DESIGN FOR LEARNING (UGL): MATHS FOR ALL

1. multiple means of **representation** (SDL)

Make invisible visible

1. multiple means for **action and expression** (SCL)

Make unfamiliar familiar

2. multiple means for **engagement** (MPL)

Make indifferent different

(See Rose & Gravel's Universal Design for Learning. In B. McGaw, P. Peterson, & E. Baker (Eds.), *International encyclopedia of education*, 3rd ed., pp. 119-124, 2010)

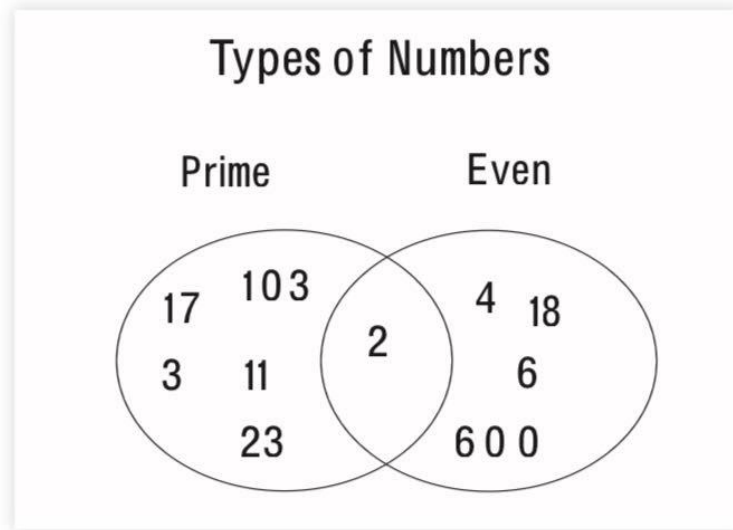
CLASSROOM INTER-ACTIONS

Teaching and learning is a 'joint labour'.
It requires both teachers and students to
have joint efforts at taking

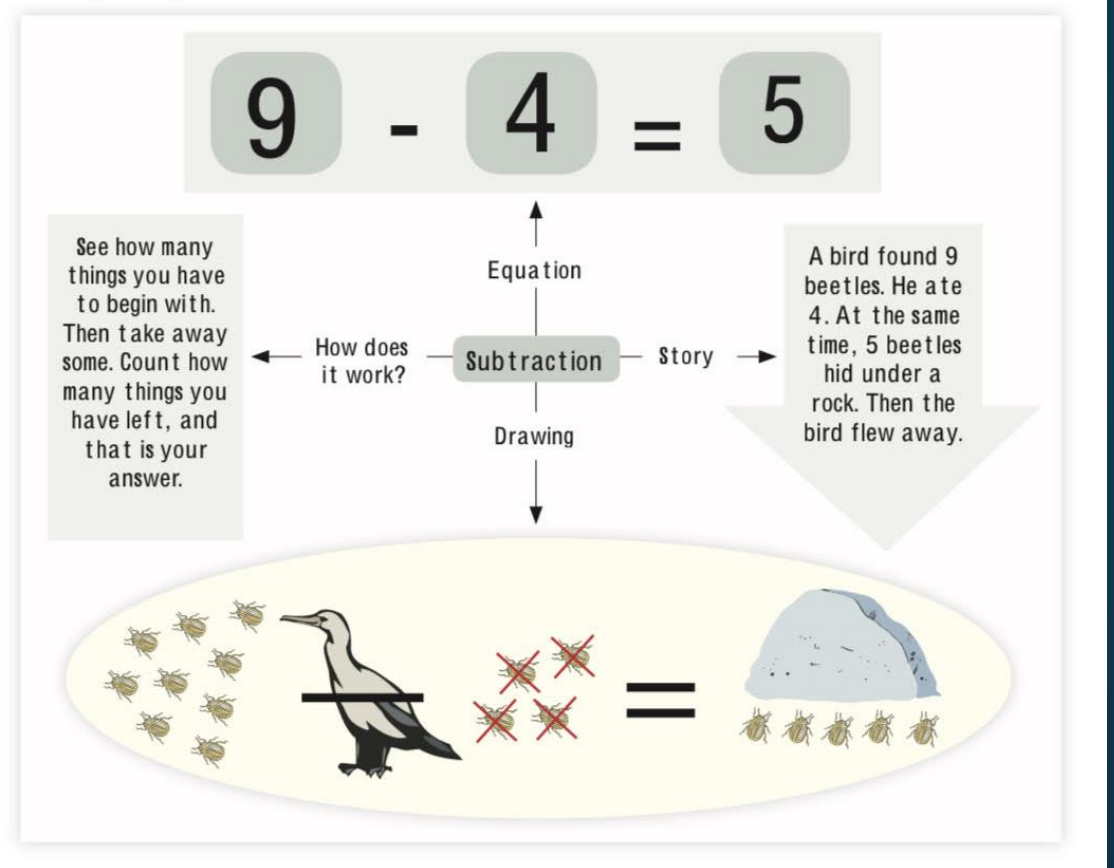
ACTIONS to QUESTION!

LEARNING LOG: VISUALISE THE CONCEPTS

Venn Diagram



Concept Map



(Source: Vicki Urquhart's *Using Writing in Mathematics to Deepen Student Learning*, 2009)

HUNDREDS CHART

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

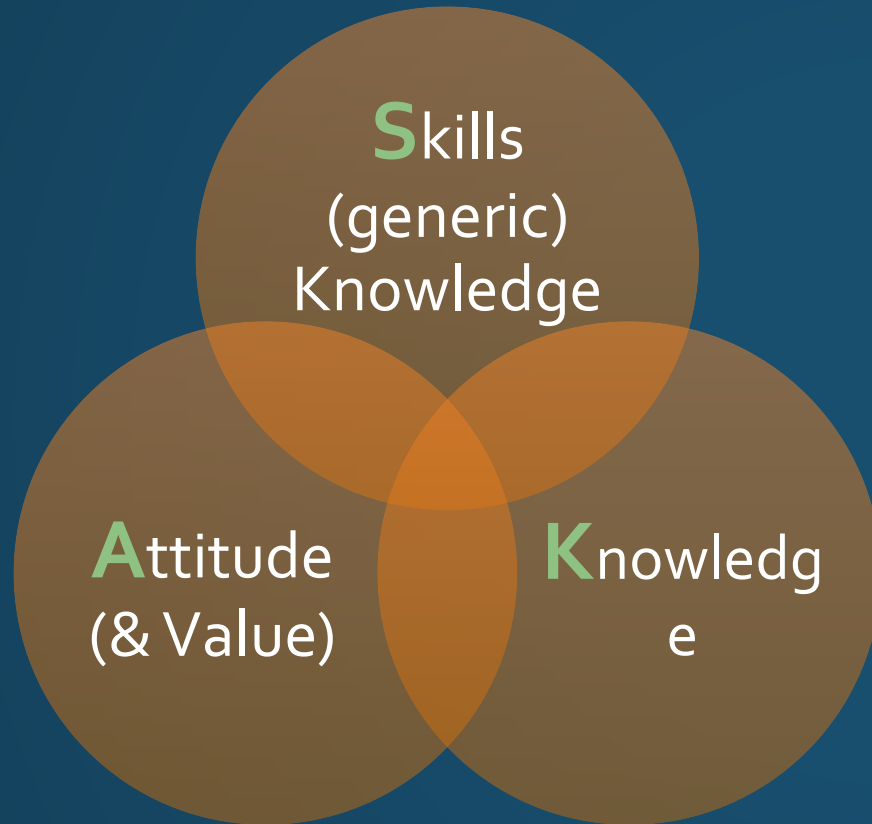
VISUALISE MYSTERY NUMBER: THE HUNDREDS CHART

1. What number is one above 83 (\uparrow)?
2. What number is three below 65 ($\downarrow\downarrow\downarrow$)?
3. What number is two to the left and one above 40 ($\leftarrow\leftarrow\uparrow$)?
4. What number is 33 ($\uparrow\rightarrow\rightarrow$)?
5. 72 $\uparrow\uparrow\rightarrow$
6. 85 $\downarrow\rightarrow\rightarrow$
7. 33 $\uparrow\uparrow\rightarrow\rightarrow$
8. 45 $\uparrow\uparrow\rightarrow\downarrow\rightarrow$
9. 1 $\rightarrow\rightarrow\downarrow\downarrow\downarrow\downarrow$
10. 74 $\rightarrow\downarrow\downarrow\rightarrow\rightarrow\rightarrow$
11. 4 $\downarrow\downarrow\downarrow\rightarrow$
12. 30 $\uparrow\leftarrow\leftarrow\downarrow\downarrow$

Coffee Break



PRIMARY MATHEMATICS: ASK & STRANDS



Five **Strands** (KS1 & KS2):

1. Number
2. Algebra
3. Measure
4. Shape & Space
5. Data Handling

Revised Bloom Questioning Techniques



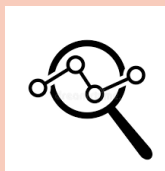
REMEMBER:
What is perimeter? What is area?



UNDERSTAND:
What are the perimeters and areas of these two shapes?



APPLY: How do the perimeters and areas of these two shapes compare?



ANALYSE: If you increase the width of a rectangle by 2 units, what happens to the area and perimeter?



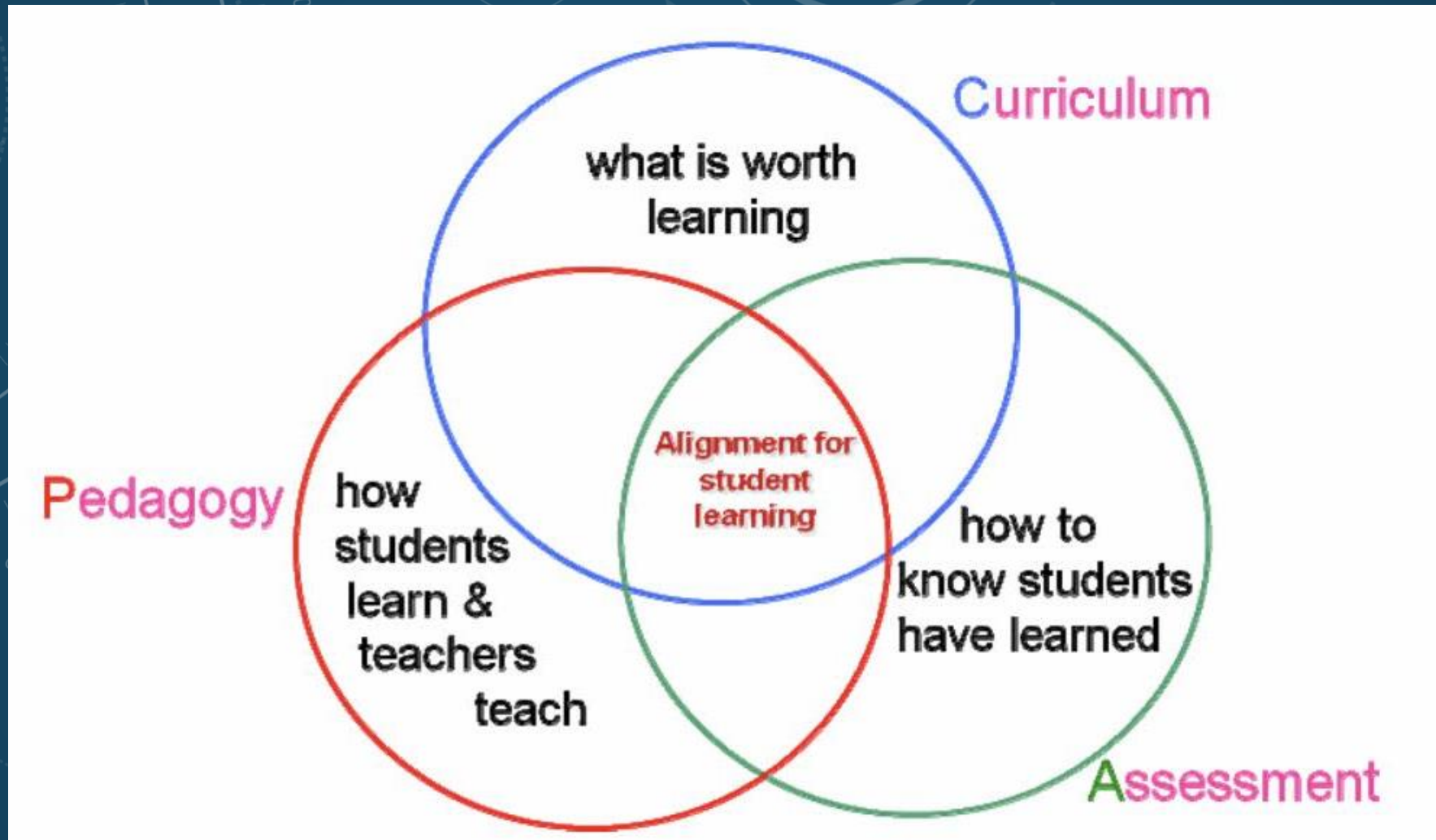
EVALUATE:
Which floor plan provides a larger seating capacity? Why?



CREATE: How would you design a floor plan that meets the needs of your client?

How to ask?

ALIGNMENT FOR STUDENT LEARNING



(WARDLAW, 2008)

STUDENT SELF-EVALUATION

Self-evaluation by students: Tick one symbol in each section.

1	I could understand the content of this lesson.	<input checked="" type="radio"/> <input type="radio"/> <input type="triangle"/>
2	I could make use of ideas that I had already learned.	<input checked="" type="radio"/> <input type="radio"/> <input type="triangle"/>
3	I could explain my own ideas to other students.	<input checked="" type="radio"/> <input type="radio"/> <input type="triangle"/>

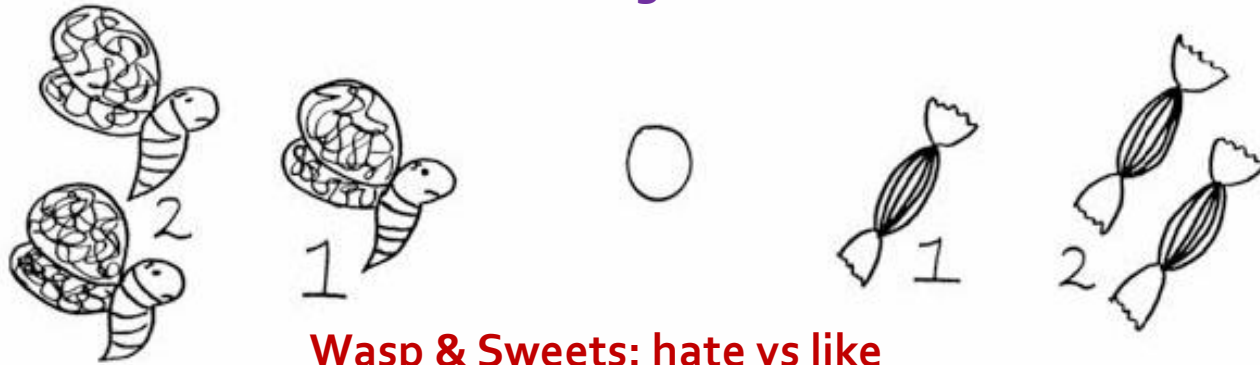
Fully understood

Understood but not yet confident

Not too sure yet



Ticks & Crosses: good vs bad



Wasp & Sweets: hate vs like



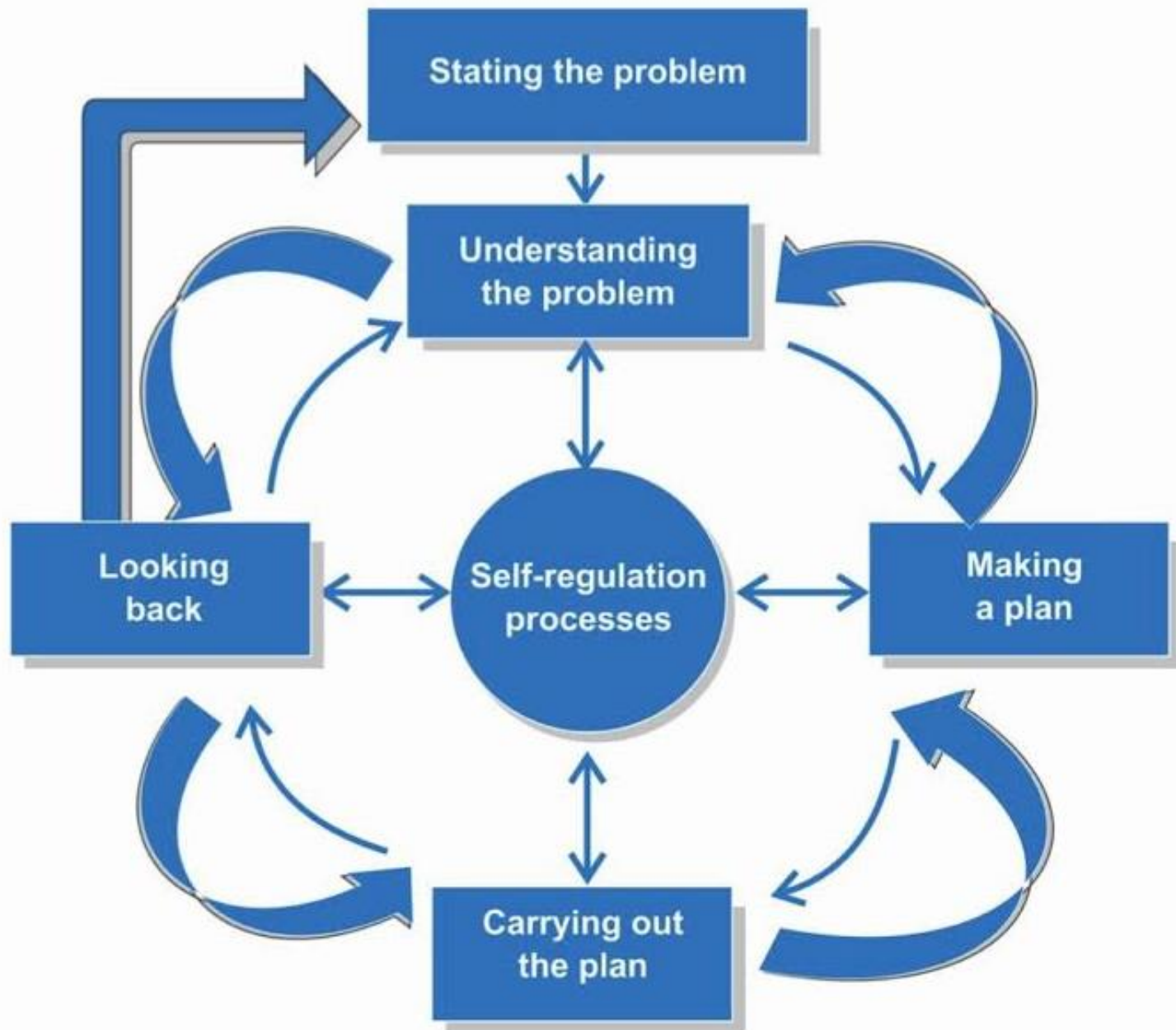
Anxiety: Mr/Ms happy vs Mr/Ms worry



Unhappiness at failure: frowny vs smiley

PICTORIAL RATING SCALE: MATHEMATICS ATTITUDE AND ANXIETY

(DOWKER, CHERITON, HORTON & MARK'S RELATIONSHIPS BETWEEN ATTITUDES AND PERFORMANCE IN YOUNG CHILDREN'S MATHEMATICS, 2019)



POLA'S PROBLEM SOLVING CYCLIC MODEL

(WILSON, FERNANDEZ & HADAWAY, 1994)

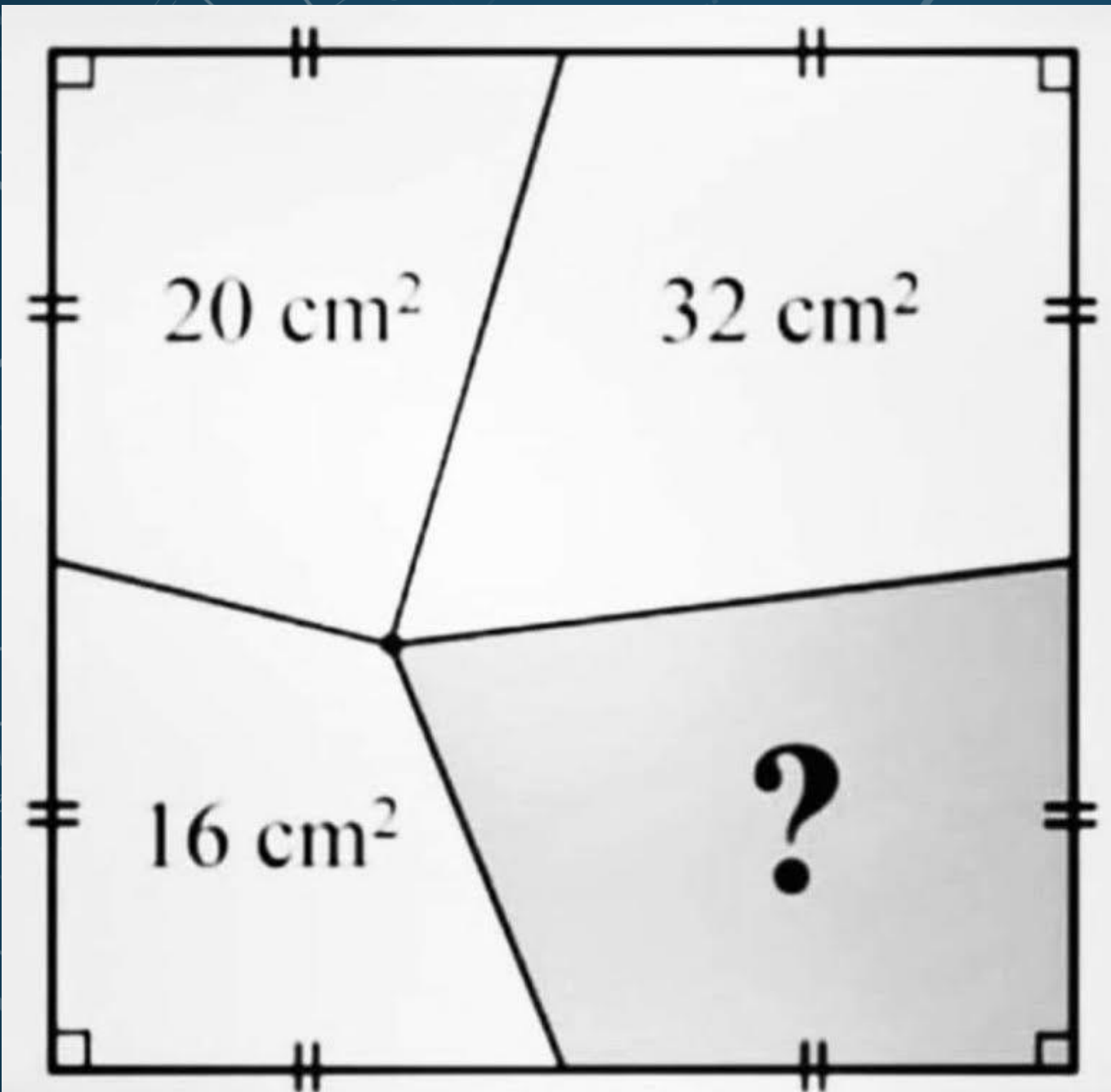
GROUP ACTIVITY TASK (1): 蝸牛爬出洞穴問題

洞穴的深度：15 米

日間：向上爬 $7\frac{2}{5}$ 米

晚間：向下滑 $3\frac{3}{10}$ 米

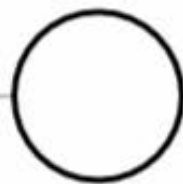
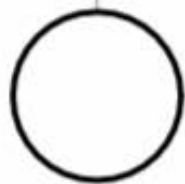
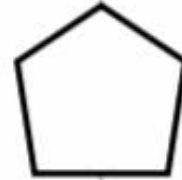
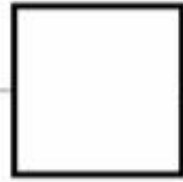
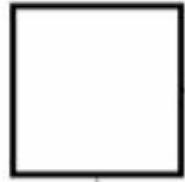
◆ 求解：第幾天蝸牛會爬出洞穴？



GROUP ACTIVITY
TASK (2): THE
MISSING AREA

GROUP ACTIVITY TASK (3): SHAPE & NUMBER

WHAT NUMBER IS EACH SHAPE?



The Same shape stands for the same number

Each row or column add up to 9

Zero is not allowed

GROUP ACTIVITY (4): BALANCE PUZZLE

THERE ARE 9 COINS, ALL EXCEPT ONE ARE THE SAME WEIGHT, THE ODD ONE IS HEAVIER THAN THE REST. YOU MUST DETERMINE WHICH IS THE ODD ONE OUT USING AN OLD FASHIONED BALANCE. YOU MAY USE THE BALANCE TWICE. EXPLAIN HOW THIS CAN BE DONE.

解難例題：蝸牛爬出洞穴問題

列出相關資料

◆ 已知條件

洞穴的深度：**15** 米

日間：向上爬 $7\frac{2}{5}$ 米

晚間：向下滑 $3\frac{3}{10}$ 米

◆ 求解

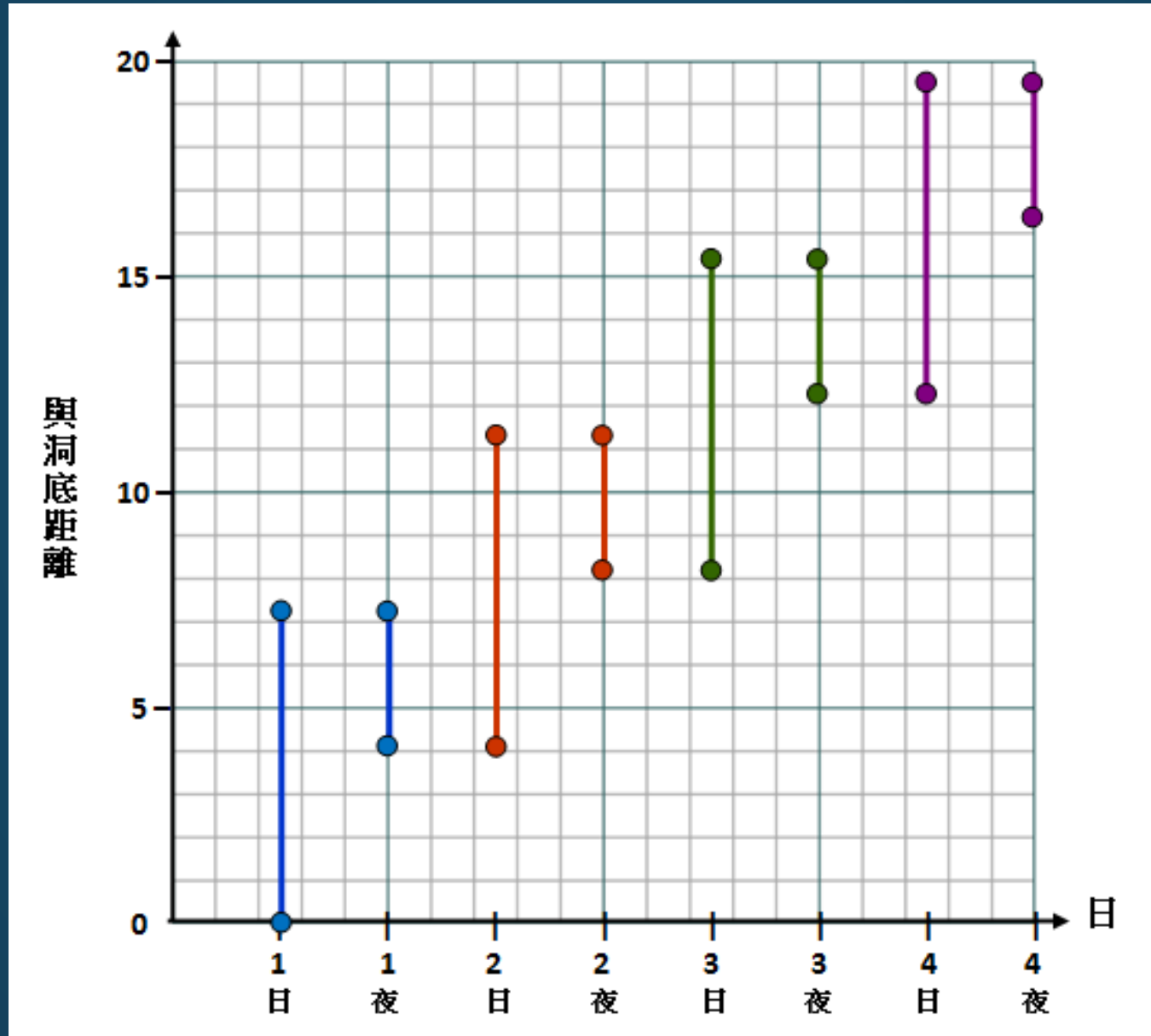
第幾天蝸牛會爬出洞穴

*答案不能是：第 **2.5**天；第 **3.2**天，...

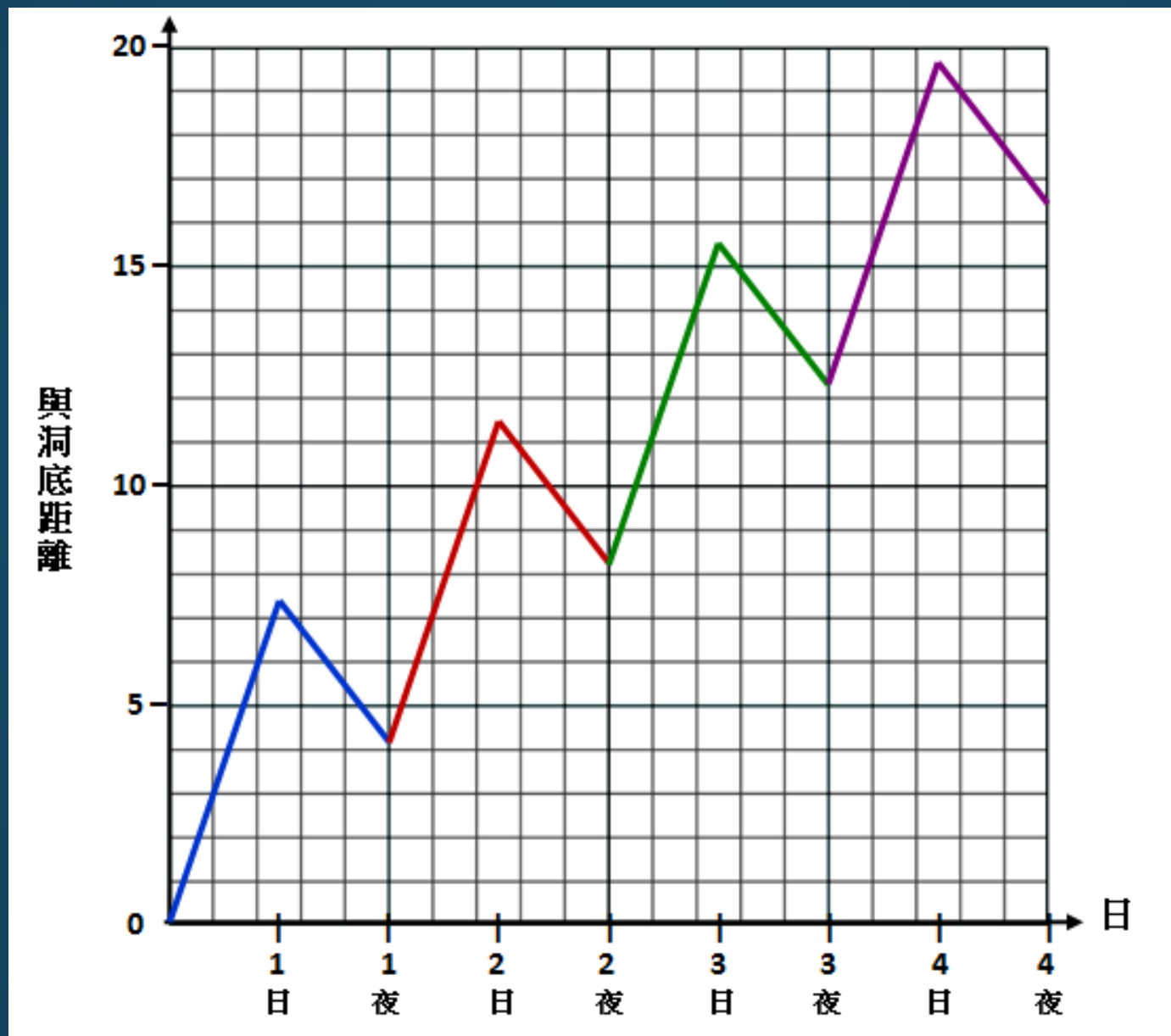
➤ 可行方法：列表

日 數		計 算	與洞底距離 (米)
1	日		
	夜		
2	日		
	夜		
3	日		
	夜		
4	日		
	夜		
.			

可行方法：繪圖



可行方法：圖表



可行方法：建立公式

若已知條件為

- (1)** 洞穴的深度：**H** 米
- (2)** 日間：向上爬 **D**米
- (3)** 晚間：向下滑 **N**米

可行方法：建立公式

建立公式的構想

- (a)** 離開洞穴一天前所需爬行的距離
- (b)** 每日（經上爬及下滑）爬行的距離
- (c)** 離開洞穴一天前所需爬行的日數
（要進位至最大的整數）
- (d)** 離開洞穴所需爬行的日數

可行方法：建立公式

建立公式

- (a) 離開洞穴一天前所需爬行的距離： **$H - D$**
- (b) 每日（經上爬及下滑後）爬行的距離： **$D - N$**
- (c) 離開洞穴一天前所需爬行的日數（要進位至最大的整數）：
 $(H - D) \div (D - N)$
- (d) 離開洞穴所需爬行的日數：
 $(H - D) \div (D - N) + 1$

- ◆ 若不分步計算而運用最後的公式，則於得出最後的答案時要進位至最大的整數

延伸問題

有兩隻（或多於兩隻）的蝸牛同時掉進洞穴，每隻蝸牛每日向上爬及向下滑的距離均有所不同。
如何決定那一隻蝸牛會最先離開洞穴？

The background is a dark blue gradient with several faint, light blue technical diagrams. These include circular gauges with numerical scales (e.g., 150, 160, 170, 180, 190, 200, 210, 220, 230, 240, 250, 260) and various circular arrows indicating clockwise or counter-clockwise rotation. The overall aesthetic is technical and precise.

CHANGE ONE WORD AND CHANGE THE MEANING AS A WHOLE

The man walked into the house

LESSON STUDY:

STAR APPROACH FOR PREPARING LESSON

Situation (Classroom milieu?)

Task (Rich task?)

Action (Flipping questions?)

Reflect (Got improving!?)

WORKSHOP ACTIVITY

- Task 1: Write a Math Lesson Plan using Bloom Questioning Techniques**
- Task 2: Design some flip questions in preparing a lesson**

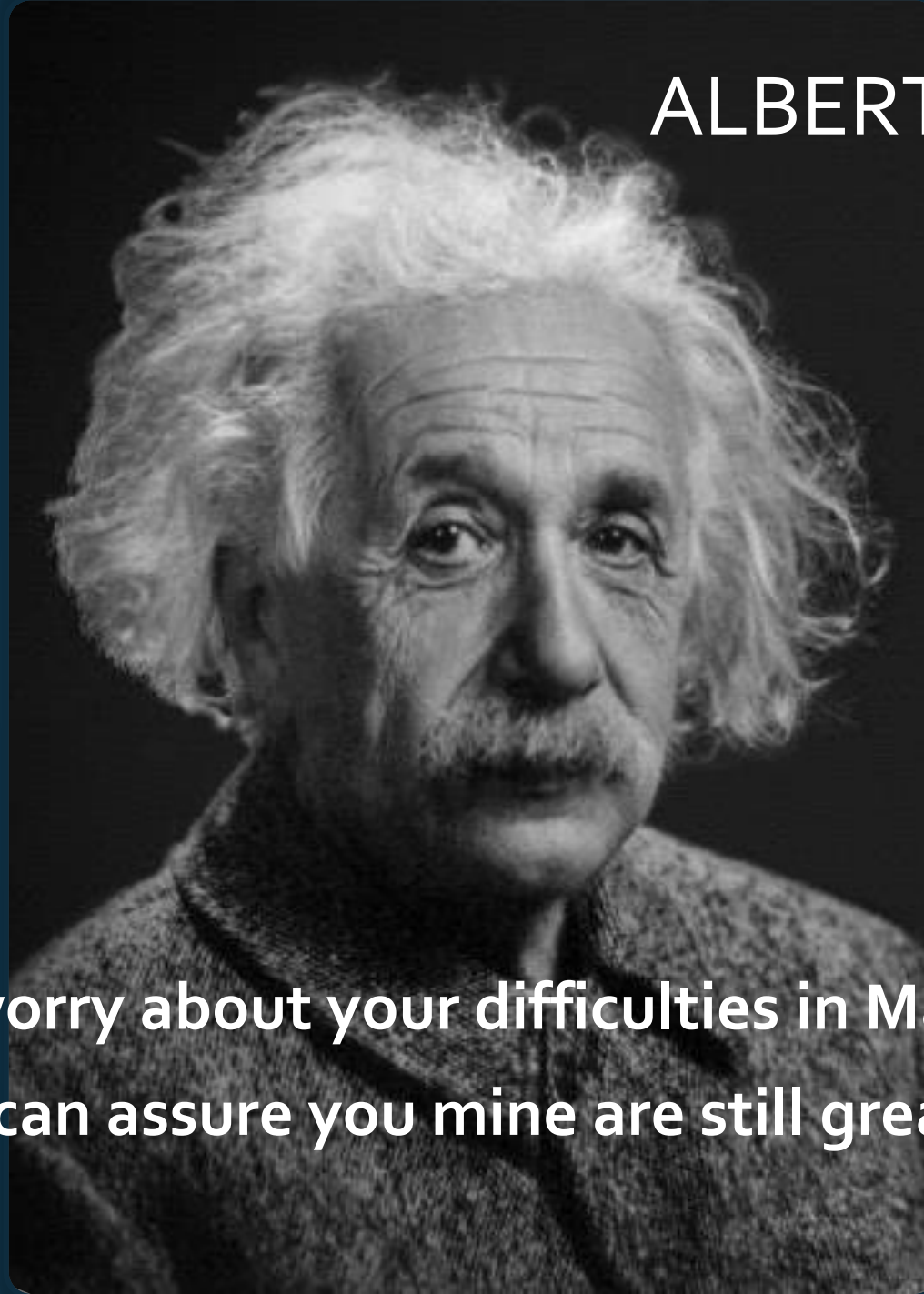
深層的「4P」

一個具備良好數學素養的人 (**P**erson) 會有更大的潛力 (**P**otentiality) 通過不同的視野觀點 (**P**erspective) 為未來帶來不一樣的想像可能 (**P**ossibility)。

在學生成長旅途上，但願看到他們從尋覓數學意義過程，能享受到那份難得真趣。

LAW, H.Y. (2022). MATHEMATICAL LITERACY: IN SEARCH OF MATHEMATICAL MEANING THROUGH LEARNING EXPERIENCES. *SCHOOL MATHEMATICS NEWSLETTER, ISSUE 25*, EDB, HKSAR.

ALBERT EINSTEIN



**Do not worry about your difficulties in Mathematics.
I can assure you mine are still greater.**